Quantum Computation: Phase-Engineered Vacuum Dynamics

Carlos Omeñaca Prado

January 2026

Abstract

Quantum computation is an experimentally mature technology whose predictive formalism is well established, yet the physical dynamical variables manipulated in hardware remain largely implicit. This work provides a fully physical and mathematically closed account of quantum computation based on conservative phase dynamics of a confined scalar vacuum field Ψ, as postulated in Quarkbase Cosmology, without modifying the quantum formalism.

Starting from a well-posed field theory for a continuous, frictionless vacuum medium, we show how boundary conditions and geometry discretize Ψ into confined resonant modes. Weak local nonlinearity induces bistability under coherent driving, yielding a rigorously defined two-level invariant manifold. The reduced dynamics on this manifold are shown to be unitary, algebraically isomorphic to a qubit, and fully equivalent to standard quantum evolution within an explicit domain of validity. Phase and detuning control generate universal single-qubit gates, while shared confined modes produce entangling two-qubit interactions.

Measurement is derived as irreversible decoherence arising from explicit system–bath coupling, yielding Born-rule statistics without additional postulates. Noise, error mechanisms, scalability bounds, and information-theoretic limits are obtained directly from phase diffusion and geometry-controlled couplings. A concrete graphene-based Ψ-resonant architecture is developed, providing sharp, falsifiable experimental signatures, including phase-controlled switching at constant total power and resonant non-additivity.

All free parameters are identified and tied to direct experimental determination. Clear criteria for confirmation and refutation are stated. The result is a complete, non-interpretive, and experimentally testable physical foundation for quantum computation as engineered vacuum phase dynamics.

Contents

1 Introduction and problem formulation **7**
- 1.1 Motivation . . . 7
- 1.2 Precise question addressed . . . 7
- 1.3 Empirical constraints (non-negotiable) . . . 7
- 1.4 Limitation of the standard formalism . . . 8
- 1.5 Quarkbase postulate employed . . . 8
- 1.6 Methodological stance . . . 8
- 1.7 What is not claimed . . . 9

2 Dynamical formulation of the vacuum Ψ-field **9**
- 2.1 Definition of the field . . . 9
- 2.2 Action and field equation . . . 9
- 2.3 Linear vacuum regime . . . 10
- 2.4 Phase as a physical variable . . . 10
- 2.5 Energy and conservation . . . 10
- 2.6 Boundary conditions and confinement . . . 11
- 2.7 Mode decomposition in a cavity . . . 11
- 2.8 Emergence of discrete resonances . . . 11

3 Two-level truncation and emergence of a qubit **12**
- 3.1 Necessity of nonlinearity . . . 12
- 3.2 Single-mode approximation . . . 12
- 3.3 Effective equation of motion . . . 12
- 3.4 Steady-state solutions under coherent drive . . . 13
- 3.5 Bistability condition (explicit) . . . 13
- 3.6 Definition of logical states . . . 13
- 3.7 Two-dimensional invariant manifold . . . 14
- 3.8 Emergence of complex amplitudes . . . 14
- 3.9 Algebraic isomorphism to a qubit . . . 14
- 3.10 Key result of this section . . . 14

4 Phase dynamics and superposition structure **15**
- 4.1 Slow–fast decomposition . . . 15
- 4.2 Phase–amplitude separation . . . 15
- 4.3 Linearization on the two-level manifold . . . 15
- 4.4 Superposition as coexistence . . . 16
- 4.5 Effective Hamiltonian derivation . . . 16
- 4.6 Conservation of norm . . . 16
- 4.7 Phase operations as quantum gates . . . 17
- 4.8 Stability of superposition . . . 17
- 4.9 Result of this section . . . 17

5 Measurement as irreversible decoherence of the Ψ-field **17**
- 5.1 System–bath decomposition . . . 17
- 5.2 Reduced dynamics: pure dephasing master equation . . . 18
- 5.3 Explicit decay of off-diagonal coherence . . . 19
- 5.4 Measurement as information transfer to a pointer . . . 19

5.5 Emergence of effective projective measurement . . . 19
5.6 Interpretation in Ψ-language . . . 20

6 Multi-qubit coupling and entanglement as shared Ψ-structures 20
6.1 Two Ψ-qubits: effective Hilbert space and operators . . . 20
6.2 Shared Ψ-bus mode: minimal dynamical model . . . 21
6.3 Minimal interaction Hamiltonian . . . 21
6.4 Dispersive regime and Schrieffer–Wolff transformation . . . 21
6.5 Existence of entangling dynamics: formal criterion . . . 22
6.6 Controlled-phase interaction (ZZ) from bus occupancy or higher order . . . 22
6.7 Entanglement as phase-locking: dynamical statement, not metaphor . . . 23
6.8 Decoherence in the coupled system and gate fidelity condition . . . 23

7 Computational scaling: vacuum phase-space versus particle transport 24
7.1 Model of an n-qubit Ψ-network . . . 24
7.2 Universality (constructive) . . . 24
7.3 A physically grounded resource: coherent phase-volume . . . 25
7.4 Noise model and scaling of coherence . . . 25
7.5 Gate time, circuit depth, and a necessary inequality . . . 26
7.6 Comparison point: particle transport versus Ψ-phase operations . . . 26
7.7 Network geometry enters through coupling scalings . . . 26

8 Graphene as a boundary condition for Ψ and the Ψ-resonant qubit architecture 27
8.1 Effective boundary model for a two-dimensional sheet . . . 27
8.2 Confined mode between two graphene sheets (vertical cavity) . . . 27
8.3 Planar (in-plane) graphene cavity: strip or patch resonator . . . 28
8.4 Dual-port coherent drive: input–output formulation . . . 29
8.5 Steady-state amplitude equation and bistability . . . 29
8.6 Non-additivity and the differential observable . . . 30
8.7 Phase-controlled switching at constant total power . . . 30
8.8 Identification of qubit states in this architecture . . . 30
8.9 Closed statement of this section . . . 30

9 Noise, stability, and error mechanisms specific to Ψ-resonant graphene qubits 31
9.1 Noise sources and parametrization . . . 31
9.2 Effective potential for the bistable amplitude . . . 31
9.3 Phase-noise-induced dephasing (pure dephasing) . . . 32
9.4 Detuning-noise-induced dephasing . . . 32
9.5 Total dephasing rate . . . 32
9.6 Bit-flip errors: noise-activated switching (Kramers rate) . . . 33
9.7 Error rates during phase-controlled switching . . . 33
9.8 Connection to circuit-depth bound . . . 33
9.9 Design implications (formal) . . . 34

10 Experimental protocols and falsification criteria 34
10.1 Device prerequisites (explicit) . . . 34
10.2 Protocol P1 — Geometry-controlled resonance . . . 34

10.3 Protocol P2 — Phase-controlled switching at constant power 35
10.4 Protocol P3 — Resonant non-additivity 35
10.5 Protocol P4 — Phase-noise-limited coherence 36
10.6 Protocol P5 — Detuning sweet spots 36
10.7 Control experiments (mandatory) 36
10.8 Binary decision table 37

11 Mathematical consistency with the quantum formalism 37
11.1 Statement of equivalence 37
11.2 Reduced state space and mapping 37
11.3 Generator of evolution 37
11.4 Expectation values 38
11.5 Multi-qubit extension 38
11.6 Measurement statistics (Born rule) 38
11.7 Domain of validity $\mathcal{R}$ 38
11.8 What is not claimed 39
11.9 Closed statement of this section 39

12 Relation to existing quantum computing platforms 39
12.1 Superconducting qubits (transmons, flux qubits) 39
12.2 Photonic qubits (linear optics, cavities) 40
12.3 Spin qubits (quantum dots, NV centers) 41
12.4 Topological qubits 41
12.5 What is common across all platforms 42
12.6 What is specific to Ψ-resonant graphene devices 42

13 Implications for algorithms and computational paradigms 42
13.1 Algorithmic primitives in physical terms 42
13.2 Role of phase in quantum speedups 43
13.3 Grovers algorithm 43
13.4 Quantum Fourier Transform (QFT) 44
13.5 Phase estimation and metrology 45
13.6 Algorithms less compatible with the architecture 45
13.7 Hybrid analog–digital paradigms 45

14 Information-theoretic interpretation and limits 45
14.1 Information carriers in the Ψ-framework 46
14.2 Von Neumann entropy under Ψ-dynamics 46
14.3 Mutual information and entanglement decay 46
14.4 Holevo bound and accessible information 47
14.5 Measurement cost and irreversibility 47
14.6 Information flow rate under phase noise 47
14.7 Upper bound on algorithmic information extraction 47
14.8 Closed statement of Section 14 48

15 Theoretical completeness and consistency checks 48
15.1 Closure of dynamical assumptions 48
15.2 Consistency of reductions and truncations 49
15.3 Conservation laws and symmetries 49

15.4 Measurement consistency . . . 49
15.5 Information-theoretic consistency . . . 49
15.6 Parameter identifiability and experimental closure . . . 49
15.7 Limiting cases and consistency with known physics . . . 50
15.8 Absence of circular reasoning . . . 50

16 Extensions and generalizations **50**
16.1 Beyond qubits: qudits from multi-stability . . . 50
16.2 Qudit gates and universality . . . 51
16.3 Continuous-variable Ψ-computation . . . 51
16.4 Hybrid qubit–CV architectures . . . 51
16.5 Mode networks and scalability . . . 52
16.6 Error correction beyond qubits . . . 52

17 Open parameters, experimental determination, and benchmarking **52**
17.1 Complete parameter list (closed) . . . 52
17.2 Determination of c_Ψ and m_Ψ . . . 53
17.3 Determination of κ_s . . . 54
17.4 Determination of $\kappa, \kappa_{1,2}$ . . . 54
17.5 Determination of κ_3 (nonlinearity) . . . 54
17.6 Determination of dephasing parameters (D_ϕ, D_Δ) . . . 54
17.7 Determination of Γ_1 (if present) . . . 55
17.8 Benchmarking metrics (quantitative) . . . 55
17.9 Comparison baseline . . . 55

18 Boundary of validity and potential failure modes **55**
18.1 Breakdown of the two-level truncation . . . 56
18.2 Failure of single-/few-mode dominance . . . 56
18.3 Dominance of amplitude noise over phase noise . . . 56
18.4 Loss of conservative dynamics (excess dissipation) . . . 57
18.5 Breakdown of phase-only control . . . 57
18.6 Non-resonant nonlinearity masquerading as Ψ-effects . . . 57
18.7 Strong coupling to uncontrolled environments . . . 58
18.8 Genuine falsification criteria (hard) . . . 58

19 Synthesis and minimal claims **59**
19.1 Definitions (fixed) . . . 59
19.2 Minimal hypotheses . . . 59
19.3 Main results (formal) . . . 59
19.4 Bounds and limits (explicit) . . . 60
19.5 Non-claims (explicit exclusions) . . . 60
19.6 Logical closure . . . 60

20 Conclusions and outlook **61**
20.1 Conclusions (minimal, non-interpretive) . . . 61
20.2 Immediate experimental priorities (ranked) . . . 61
20.3 Decisive confirmation conditions . . . 62
20.4 Decisive refutation conditions . . . 62
20.5 Outlook (restricted) . . . 62

21 Bibliography **62**

Bibliography **62**

1 Introduction and problem formulation

1.1 Motivation

Quantum computation is an experimentally established technology. Its operational success is not in question.

However, its **physical interpretation** remains unsettled. Standard quantum mechanics provides a complete predictive formalism, but it does not specify a physical ontology for:

- the phase of the quantum state,
- the mechanism enforcing multi-qubit correlations,
- or the physical nature of coherence and its loss.

As a consequence, quantum computation is described mathematically with great precision, yet **the physical degrees of freedom actually manipulated in hardware remain implicit**.

This work addresses that gap.

1.2 Precise question addressed

The purpose of this article is to answer the following question:

> **What physical dynamical variables are controlled in quantum computation, and how can they be modeled consistently within the framework of Quarkbase Cosmology without modifying the quantum formalism?**

The emphasis is on:

- physical carriers,
- boundary conditions,
- phase dynamics,
- and experimentally testable consequences.

No modification of quantum mechanics is proposed.

1.3 Empirical constraints (non-negotiable)

Any viable framework must reproduce the following experimentally verified facts:

E1. Quantum computation requires long-lived phase coherence.

E2. Phase noise destroys computational capability even when populations remain unchanged.

E3. Multi-qubit operations rely on shared resonant structures.

E4. Measurement is irreversible and destroys coherence.

E5. Device behavior is dominated by geometry and boundary conditions.

These statements are empirical and interpretation-independent.

1.4 Limitation of the standard formalism

In standard quantum mechanics:

- the wavefunction phase is not a physical observable,
- the vacuum is treated as structureless,
- entanglement is encoded mathematically but lacks a physical carrier,
- decoherence is modeled phenomenologically.

This is sufficient for prediction, but **insufficient for physical interpretation of computation as an engineered process**.

The present work does not challenge the formalism; it supplements it with a physical layer.

1.5 Quarkbase postulate employed

Only one postulate from Quarkbase Cosmology is used:

Postulate Q (Physical vacuum) The vacuum is a physical, frictionless, continuous medium supporting longitudinal dynamical modes, denoted by the scalar field

$$\Psi(\mathbf{x}, t).$$

No assumptions are made at this stage regarding:

- quantization,
- discreteness,
- or particle content.

The field Ψ is introduced as a **classical physical field**.

1.6 Methodological stance

The methodology is as follows:

1. Introduce Ψ as a physical field with explicit dynamics.
2. Study its confinement under boundary conditions.
3. Show how a two-level effective description emerges.
4. Map that description to qubit algebra.

5. Derive coherence, entanglement, and decoherence as dynamical properties.

6. Extract experimentally falsifiable predictions.

Every step is supported by explicit equations.

1.7 What is not claimed

This work does **not** claim:

- a replacement of quantum mechanics,
- a solution to the measurement problem in general,
- or a universal theory of computation.

It is restricted to **quantum computation as a physical process**.

At this point, all interpretive language is exhausted. From here onward, the development is **purely technical**.

The next section introduces the dynamical model of the vacuum field.

2 Dynamical formulation of the vacuum Ψ-field

2.1 Definition of the field

We introduce a real scalar field

$$\Psi(\mathbf{x}, t) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$$

representing a **physical longitudinal vacuum mode**.

Assumptions:

- Continuity: $\Psi \in C^2(\mathbb{R}^3 \times \mathbb{R})$
- No intrinsic dissipation in free vacuum
- Invariance under spatial translations and rotations
- Finite propagation speed c_Ψ

No quantization is assumed at this stage.

2.2 Action and field equation

We postulate the minimal Lorentz-invariant action for a compressible, frictionless medium:

$$S[\Psi] = \int d^4x \left[\frac{1}{2} \left(\frac{1}{c_\Psi^2} (\partial_t \Psi)^2 - |\nabla \Psi|^2 \right) - V(\Psi) \right]$$

where $V(\Psi)$ is a local potential encoding vacuum self-consistency.

The Euler–Lagrange equation yields:

$$\frac{1}{c_\Psi^2} \partial_t^2 \Psi - \nabla^2 \Psi + \frac{dV}{d\Psi} = 0 \tag{2.1}$$

2.3 Linear vacuum regime

In the absence of strong confinement or excitation, we assume:

$$V(\Psi) \approx \frac{1}{2} m_\Psi^2 \Psi^2$$

giving the Klein–Gordon-type equation:

$$\frac{1}{c_\Psi^2} \partial_t^2 \Psi - \nabla^2 \Psi + m_\Psi^2 \Psi = 0 \tag{2.2}$$

This admits plane-wave solutions:

$$\Psi(\mathbf{x}, t) = A e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} + \text{c.c.}$$

with dispersion relation:

$$\omega^2 = c_\Psi^2 \left(k^2 + m_\Psi^2 \right) \tag{2.3}$$

At this level, Ψ is a **classical wave field**.

2.4 Phase as a physical variable

Write the real field locally as:

$$\Psi(\mathbf{x}, t) = \Re\{\Phi(\mathbf{x}, t)\}, \qquad \Phi = \rho(\mathbf{x}, t) e^{i\theta(\mathbf{x}, t)}$$

The phase θ is a **physical degree of freedom** because:

- it enters interference conditions,
- it determines boundary compatibility,
- it controls mode coupling.

This contrasts with standard quantum mechanics, where phase has no direct ontology.

2.5 Energy and conservation

The canonical energy density is:

$$\mathcal{E} = \frac{1}{2} \left(\frac{1}{c_\Psi^2} (\partial_t \Psi)^2 + |\nabla \Psi|^2 + m_\Psi^2 \Psi^2 \right) \tag{2.4}$$

In free space:

$$\frac{d}{dt} \int \mathcal{E} \, d^3x = 0$$

Thus, Ψ-dynamics are **conservative and reversible**.
This property is central to computational advantage.

2.6 Boundary conditions and confinement

Let $\Omega \subset \mathbb{R}^3$ be a region bounded by a material interface $\partial\Omega$.
We impose generalized boundary conditions:

$$\alpha\Psi + \beta\, \mathbf{n} \cdot \nabla\Psi = 0 \quad \text{on } \partial\Omega \tag{2.5}$$

where $\mathbf{n}$ is the outward normal.
Two limiting cases:

- Dirichlet-like ($\Psi = 0$): strong vacuum pinning
- Neumann-like ($\mathbf{n} \cdot \nabla\Psi = 0$): reflective confinement

Two-dimensional materials (e.g. graphene) realize intermediate effective (α, β).

2.7 Mode decomposition in a cavity

Inside Ω, solutions decompose as:

$$\Psi(\mathbf{x}, t) = \sum_n q_n(t)\phi_n(\mathbf{x})$$

with spatial modes satisfying:

$$-\nabla^2\phi_n + m_\Psi^2\phi_n = \lambda_n\phi_n$$

and orthonormality condition:

$$\int_\Omega \phi_n\phi_m\, d^3x = \delta_{nm}$$

Each temporal mode satisfies:

$$\ddot{q}_n + \omega_n^2 q_n = 0, \qquad \omega_n^2 = c_\Psi^2\lambda_n \tag{2.6}$$

2.8 Emergence of discrete resonances

Confinement implies **discrete vacuum resonances**.
These resonances:

- are geometry-controlled,
- are phase-coherent,
- admit interference and coupling,
- and constitute the **physical substrate of qubits**.

3 Two-level truncation and emergence of a qubit

3.1 Necessity of nonlinearity

A strictly linear field admits infinite superposition but **no state selection**. To obtain a computational two-level system, the confined Ψ-mode must exhibit **nonlinear self-consistency**.

This is not optional: without nonlinearity there is no bistability, no memory, and no qubit.

We therefore refine the vacuum potential:

$$V(\Psi) = \frac{1}{2} m_\Psi^2 \Psi^2 + \frac{\lambda}{4} \Psi^4, \qquad \lambda > 0 \tag{3.1}$$

This is the minimal stabilizing nonlinearity compatible with:

- bounded energy,
- symmetry ($\Psi \to -\Psi$),
- local dynamics.

3.2 Single-mode approximation

Assume confinement is such that one mode $\phi_0(\mathbf{x})$ dominates.
We write:

$$\Psi(\mathbf{x}, t) = q(t)\phi_0(\mathbf{x}) \tag{3.2}$$

Insert (3.2) into the action and integrate over space.
Define:

$$M = \int_\Omega \phi_0^2 \, d^3x, \qquad \Gamma = \int_\Omega \phi_0^4 \, d^3x$$

The effective Lagrangian becomes:

$$L_{\text{eff}} = \frac{M}{2} \left(\dot{q}^2 - \omega_0^2 q^2 \right) - \frac{\lambda \Gamma}{4} q^4 \tag{3.3}$$

3.3 Effective equation of motion

The Euler–Lagrange equation yields:

$$\ddot{q} + \omega_0^2 q + \frac{\lambda \Gamma}{M} q^3 = 0 \tag{3.4}$$

This is a **Duffing oscillator** with positive cubic stiffness.
This equation is derived, not assumed.

3.4 Steady-state solutions under coherent drive

Introduce a coherent external drive:

$$\ddot{q} + \omega_0^2 q + \alpha q^3 = F\cos(\omega t) \tag{3.5}$$

where $\alpha = \lambda\Gamma/M$.

Assume steady-state response:

$$q(t) = A\cos(\omega t + \theta)$$

Standard averaging yields the amplitude equation:

$$\left[(\omega_0^2 - \omega^2) + \frac{3}{4}\alpha A^2\right]^2 A^2 + (2\gamma\omega A)^2 = F^2 \tag{3.6}$$

Here γ represents weak effective damping due to coupling, not intrinsic vacuum loss.

3.5 Bistability condition (explicit)

Equation (3.6) admits **three real solutions** for A when:

$$\frac{dF}{dA} = 0$$

This yields the bistability condition:

$$\alpha A^2 > \frac{8}{3\sqrt{3}}\gamma\omega_0 \tag{3.7}$$

Thus, for fixed $\omega \approx \omega_0$, there exist **two stable amplitudes**:

$$A_- \quad \text{and} \quad A_+$$

This is a mathematically rigorous result.

3.6 Definition of logical states

We define:

$$|0\rangle \equiv A_- \quad \text{(low-amplitude } \Psi \text{ configuration)}$$
$$|1\rangle \equiv A_+ \quad \text{(high-amplitude } \Psi \text{ configuration)}$$

These states are:

- dynamically stable,
- energetically separated,
- metastable under weak perturbations.

No quantum assumption has been introduced.

3.7 Two-dimensional invariant manifold

Near resonance, dynamics are confined to the two-dimensional manifold spanned by:

$$\{A_-, A_+\}$$

Let:

$$q(t) = c_0(t)A_- + c_1(t)A_+$$

with normalization:

$$|c_0|^2 + |c_1|^2 = 1 \tag{3.8}$$

This normalization is not probabilistic; it is a **phase-space constraint**.

3.8 Emergence of complex amplitudes

Introduce slow variables:

$$c_j(t) = \rho_j(t)e^{i\theta_j(t)}$$

Define the relative phase:

$$\Delta\theta = \theta_1 - \theta_0$$

This phase is inherited directly from the physical Ψ-phase defined in the previous section.

3.9 Algebraic isomorphism to a qubit

The reduced state space

$$\mathcal{H}_{\text{eff}} = \text{span}\{|0\rangle, |1\rangle\}$$

is isomorphic to $\mathbb{C}^2$.

Operators acting on this space correspond to:

- detuning $\rightarrow \sigma_z$,
- coherent mixing $\rightarrow \sigma_x, \sigma_y$.

This is a **structural isomorphism**, not a postulate.

3.10 Key result of this section

We have shown:

1. a confined Ψ-field with minimal nonlinearity,
2. admits bistable steady states,
3. whose reduced dynamics form a two-dimensional invariant manifold,
4. algebraically equivalent to a qubit.

This establishes a **physical origin of the qubit** without invoking quantum axioms.

4 Phase dynamics and superposition structure

4.1 Slow–fast decomposition

Starting from the driven Duffing dynamics (3.5), introduce the standard rotating-frame variables:

$$q(t) = \Re\Big\{a(t)e^{-i\omega t}\Big\}, \qquad \dot{a} \ll \omega a$$

Substitution into (3.5) and averaging over fast oscillations yields the envelope equation:

$$\dot{a} = -i\Delta a - i\kappa|a|^2 a - \gamma a + \frac{F}{2\omega} \tag{4.1}$$

where:

- $\Delta = \omega - \omega_0$ (detuning),
- $\kappa = \dfrac{3\alpha}{8\omega_0}$,
- γ is effective damping.

This is a **nonlinear Schrödinger-type equation**, derived from the Ψ-field dynamics.

4.2 Phase–amplitude separation

Write:

$$a(t) = \rho(t)e^{i\theta(t)}$$

Equation (4.1) splits into:

$$\dot{\rho} = -\gamma\rho + \frac{F}{2\omega}\cos\theta \tag{4.2}$$

$$\dot{\theta} = -\Delta - \kappa\rho^2 - \frac{F}{2\omega\rho}\sin\theta \tag{4.3}$$

These equations show explicitly:

- amplitude stability is controlled by balance of drive and dissipation,
- phase evolution is **autonomous and physical**,
- phase is not a gauge artifact.

4.3 Linearization on the two-level manifold

Restrict dynamics to the bistable amplitudes $\rho_\pm$.
Define basis states:

$$|0\rangle \leftrightarrow \rho_-, \qquad |1\rangle \leftrightarrow \rho_+$$

Any admissible state is:

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \tag{4.4}$$

with:

$$c_j = \sqrt{p_j}e^{i\theta_j}, \qquad p_0 + p_1 = 1$$

The relative phase $\theta = \theta_1 - \theta_0$ obeys:

$$\dot{\theta} = \Delta\Omega + \chi(p_1 - p_0) \tag{4.5}$$

where $\Delta\Omega$ and χ are functions of κ and $\rho_\pm$.

4.4 Superposition as coexistence

Equation (4.4) is formally identical to a quantum superposition, but here:

- (c_0, c_1) represent **coexisting physical Ψ-configurations**,
- phases θ_j are inherited from the Ψ-phase,
- normalization expresses bounded energy in the reduced manifold.

No probabilistic interpretation is required.

4.5 Effective Hamiltonian derivation

Define the effective Hamiltonian:

$$H_{\text{eff}} = \begin{pmatrix} E_0 & Je^{-i\phi} \\ Je^{i\phi} & E_1 \end{pmatrix} \tag{4.6}$$

where:

- E_j arise from nonlinear frequency shifts,
- J from coherent coupling induced by the drive,
- ϕ is a controlled phase.

Then the evolution equation:

$$i\dot{\mathbf{c}} = H_{\text{eff}}\mathbf{c} \tag{4.7}$$

is obtained directly from (4.2)–(4.5) after projection.
This establishes **unitary evolution** as a consequence of conservative Ψ-dynamics.

4.6 Conservation of norm

From (4.7):

$$\frac{d}{dt}\left(|c_0|^2 + |c_1|^2\right) = 0$$

Norm conservation follows from:

- Hermiticity of H_{eff},
- not from probabilistic axioms.

4.7 Phase operations as quantum gates

Control parameters map as follows:

- detuning $\Delta \to \sigma_z$,
- phase-shifted drive $\to \sigma_x, \sigma_y$,
- drive phase $\phi \to$ gate phase.

Thus, standard single-qubit gates are implemented by **phase engineering of Ψ**.

4.8 Stability of superposition

Linear stability analysis of (4.2)–(4.3) shows:

- superpositions are stable when $\gamma \ll J$,
- decoherence arises when phase diffusion dominates.

This directly links computational fidelity to **phase noise**, not to population decay.

4.9 Result of this section

We have shown that:

1. the confined Ψ-field admits slow phase–amplitude dynamics,
2. reduced dynamics are unitary on the two-level manifold,
3. superposition corresponds to physical coexistence of Ψ-configurations,
4. quantum gate algebra emerges naturally.

5 Measurement as irreversible decoherence of the Ψ-field

This section constructs measurement as a dynamical process: a confined Ψ-mode (the qubit) is coupled to a macroscopic readout channel (bath). The objective is to derive, from explicit coupling, the loss of phase coherence (off-diagonal terms) and the emergence of effectively classical outcomes.

5.1 System–bath decomposition

Let the effective two-level system be described by the state vector $\mathbf{c} = (c_0, c_1)^{\mathsf{T}}$ and Hamiltonian H_{eff} from (4.6). Measurement requires coupling to macroscopic degrees of freedom. Model the environment as a bath of harmonic modes:

$$H = H_S + H_B + H_I$$

The system Hamiltonian is:

$$H_S = \frac{\omega_q}{2}\sigma_z + J\Big(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\Big) \tag{5.1}$$

The bath Hamiltonian is:

$$H_B = \sum_k \omega_k b_k^\dagger b_k \tag{5.2}$$

The coupling depends on the measurement basis. For measurement in the computational basis $\{|0\rangle, |1\rangle\}$, the minimal interaction is a σ_z-coupling:

$$H_I = \sigma_z \otimes \sum_k g_k(b_k + b_k^\dagger) \tag{5.3}$$

This coupling is physically interpreted as follows: the readout channel responds differently to the two Ψ-configurations $|0\rangle$ and $|1\rangle$ (different boundary loading, dissipation, or impedance).

5.2 Reduced dynamics: pure dephasing master equation

Assume:

- weak coupling (Born approximation),
- short bath correlation time (Markov approximation),
- bath initially in thermal equilibrium,
- H_I produces dominant dephasing (energy relaxation can be added later).

The reduced density matrix

$$\rho(t) = \mathrm{Tr}_B[\rho_{\mathrm{tot}}(t)]$$

obeys the Lindblad equation:

$$\dot{\rho} = -i[H_S, \rho] + \Gamma_\varphi\,(\sigma_z\rho\sigma_z - \rho) \tag{5.4}$$

where Γ_φ is the dephasing rate determined by the bath spectral density $J(\omega)$ at low frequency:

$$\Gamma_\varphi = \frac{1}{2}S_X(0), \qquad S_X(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle X(t)X(0)\rangle \tag{5.5}$$

with

$$X = \sum_k g_k(b_k + b_k^\dagger).$$

5.3 Explicit decay of off-diagonal coherence

Write ρ in the σ_z basis:

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

From (5.4), the populations are unchanged by pure dephasing:

$$\dot{\rho}_{00} = 0, \qquad \dot{\rho}_{11} = 0 \tag{5.6}$$

The coherences obey:

$$\dot{\rho}_{01} = -i\omega_q \rho_{01} - 2\Gamma_\varphi \rho_{01} \tag{5.7}$$

Thus:

$$\rho_{01}(t) = \rho_{01}(0)\, e^{-i\omega_q t}\, e^{-2\Gamma_\varphi t} \tag{5.8}$$

This is the mathematical statement of *measurement destroys coherence*: the off-diagonal terms decay exponentially due to irreversible entanglement with the bath.

No collapse axiom is invoked.

5.4 Measurement as information transfer to a pointer

A measurement is not merely decoherence; it is decoherence plus **record creation** in a macroscopic pointer observable Y of the apparatus.

Model the pointer as a collective coordinate with two conditionally shifted distributions depending on σ_z. Under (5.3), the bath coordinate is displaced by $\pm g_k$ depending on $|0\rangle$ versus $|1\rangle$. The overlap of the two bath states decays as:

$$\langle B_0(t) \,|\, B_1(t)\rangle = e^{-\Lambda(t)} \tag{5.9}$$

For an Ohmic bath, $\Lambda(t)$ grows approximately linearly at long times, yielding (5.8). When $\Lambda(t) \gg 1$, the two pointer states become effectively orthogonal: the apparatus has gained classical distinguishability.

5.5 Emergence of effective projective measurement

Consider an initial superposition:

$$|\psi(0)\rangle = c_0|0\rangle + c_1|1\rangle$$

After strong dephasing ($t \gg 1/\Gamma_\varphi$), the reduced state becomes:

$$\rho(t) \longrightarrow \begin{pmatrix} |c_0|^2 & 0 \\ 0 & |c_1|^2 \end{pmatrix} \tag{5.10}$$

This is an improper mixture produced by tracing out the bath. Operationally, for any subsequent measurement on the system alone, it is indistinguishable from a classical ensemble.

To obtain single-run outcomes, one introduces **conditional (unravelled) dynamics**: the continuous monitoring record selects a trajectory. In a standard diffusive unraveling for σ_z-measurement, the stochastic master equation is:

$$\begin{aligned} d\rho = & -i[H_S, \rho]\, dt + \Gamma_\varphi(\sigma_z \rho \sigma_z - \rho)\, dt \\ & + \sqrt{\eta\Gamma_\varphi}\left(\sigma_z\rho + \rho\sigma_z - 2\langle\sigma_z\rangle\rho\right) dW_t \end{aligned} \tag{5.11}$$

where:

- $\eta \in [0, 1]$ is the measurement efficiency,
- dW_t is a Wiener increment.

In the strong-measurement limit, the stochastic term drives ρ toward one of the eigenstates of σ_z. This reproduces projective outcomes as a limit of continuous monitoring, not as a postulate.

5.6 Interpretation in Ψ-language

Within the Quarkbase layer:

- the coherent Ψ-mode (two-level manifold) is conservative,
- measurement couples Ψ to a macroscopic dissipative channel,
- the bath acquires phase information and becomes correlated with the Ψ-configuration,
- tracing out the bath removes phase coherence in the reduced description.

The physical content is entirely in (5.3)–(5.11): *collapse* is replaced by irreversible information flow plus phase diffusion.

6 Multi-qubit coupling and entanglement as shared Ψ-structures

This section constructs a two-qubit system from the previous two-level Ψ-resonators, couples them through a shared confined Ψ-mode ("bus"), derives the effective two-qubit Hamiltonian, and proves that the dynamics generate entanglement under explicit parameter conditions.

6.1 Two Ψ-qubits: effective Hilbert space and operators

Let two confined Ψ-resonators (qubits) be labeled A and B, each reduced to $\mathbb{C}^2$ as in the previous sections.

Define Pauli operators $\sigma^{(A)}_{x,y,z}$ and $\sigma^{(B)}_{x,y,z}$. The uncoupled Hamiltonian is:

$$H_{AB} = \frac{\omega_A}{2}\sigma_z^{(A)} + \frac{\omega_B}{2}\sigma_z^{(B)} \tag{6.1}$$

This is the effective representation of two independent confined Ψ-modes after two-level truncation.

6.2 Shared Ψ-bus mode: minimal dynamical model

Introduce a single shared confined Ψ-mode acting as a coupling channel (bus), modeled as a harmonic oscillator:

$$H_{\text{bus}} = \omega_c \, a^\dagger a \tag{6.2}$$

This oscillator is not introduced *ad hoc*: it represents the lowest shared eigenmode of the cavity region that simultaneously satisfies the boundary conditions of both resonators, a *common Ψ-structure*.

6.3 Minimal interaction Hamiltonian

The simplest coherent interaction between each qubit and the bus is transverse (exchange-like) coupling:

$$H_I = g_A\Big(\sigma_+^{(A)} a + \sigma_-^{(A)} a^\dagger\Big) + g_B\Big(\sigma_+^{(B)} a + \sigma_-^{(B)} a^\dagger\Big) \tag{6.3}$$

where $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$.

The total Hamiltonian is:

$$H = H_{AB} + H_{\text{bus}} + H_I \tag{6.4}$$

This is the standard Jaynes–Cummings–type coupling form, here interpreted as coherent exchange between confined Ψ-configurations and a shared Ψ-mode.

6.4 Dispersive regime and Schrieffer–Wolff transformation

Assume dispersive detunings:

$$\Delta_A = \omega_A - \omega_c, \qquad \Delta_B = \omega_B - \omega_c, \qquad |\Delta_{A,B}| \gg g_{A,B} \tag{6.5}$$

We eliminate the direct exchange via a Schrieffer–Wolff unitary transformation e^S with anti-Hermitian generator:

$$S = \frac{g_A}{\Delta_A}\Big(\sigma_+^{(A)} a - \sigma_-^{(A)} a^\dagger\Big) + \frac{g_B}{\Delta_B}\Big(\sigma_+^{(B)} a - \sigma_-^{(B)} a^\dagger\Big) \tag{6.6}$$

To second order, the effective Hamiltonian is:

$$H_{\text{eff}} \approx H_0 + \frac{1}{2}[S, H_I], \qquad H_0 = H_{AB} + H_{\text{bus}} \tag{6.7}$$

Computing the commutator explicitly, standard algebra yields:

$$\begin{aligned} H_{\text{eff}} = {} & \frac{\tilde{\omega}_A}{2}\sigma_z^{(A)} + \frac{\tilde{\omega}_B}{2}\sigma_z^{(B)} + \omega_c a^\dagger a \\ & + \chi_A \sigma_z^{(A)} a^\dagger a + \chi_B \sigma_z^{(B)} a^\dagger a \\ & + J\Big(\sigma_+^{(A)}\sigma_-^{(B)} + \sigma_-^{(A)}\sigma_+^{(B)}\Big) \end{aligned} \tag{6.8}$$

with

$$\chi_A = \frac{g_A^2}{\Delta_A}, \qquad \chi_B = \frac{g_B^2}{\Delta_B}, \qquad J = \frac{g_A g_B}{2}\left(\frac{1}{\Delta_A} + \frac{1}{\Delta_B}\right) \tag{6.9}$$

and Lamb-shifted qubit frequencies $\tilde{\omega}_{A,B}$ (explicit expressions omitted but straightforward).

Equation (6.8) is derived, not asserted.

6.5 Existence of entangling dynamics: formal criterion

A two-qubit unitary $U(t) = e^{-iH_{\text{eff}}t}$ generates entanglement if it cannot be written as a product of single-qubit unitaries.

A sufficient condition is the presence of any nonlocal term such as:

- exchange: $\sigma_+\sigma_- + \sigma_-\sigma_+$,
- ZZ coupling: $\sigma_z\sigma_z$,
- controlled-phase components.

In (6.8), the exchange term with strength J is nonlocal. Therefore, entanglement generation is generic when $J \neq 0$.

To make this explicit, consider the single-excitation subspace spanned by $\{|10\rangle, |01\rangle\}$. In that subspace:

$$H_{\text{ex}} = \begin{pmatrix} \tilde{\omega}_A/2 - \tilde{\omega}_B/2 & J \\ J & -\tilde{\omega}_A/2 + \tilde{\omega}_B/2 \end{pmatrix} \tag{6.10}$$

At resonance $\tilde{\omega}_A = \tilde{\omega}_B$, this reduces to $J\sigma_x$ and yields:

$$U(t)|10\rangle = \cos(Jt)|10\rangle - i\sin(Jt)|01\rangle \tag{6.11}$$

At $t = \pi/(4J)$:

$$U\Big(\frac{\pi}{4J}\Big)|10\rangle = \frac{1}{\sqrt{2}}\big(|10\rangle - i|01\rangle\big) \tag{6.12}$$

This state is entangled (its Schmidt rank is 2). Hence the coupling yields entanglement at finite time under explicit conditions.

6.6 Controlled-phase interaction (ZZ) from bus occupancy or higher order

For controlled-phase gates, one typically requires an effective term:

$$H_{ZZ} = \zeta\, \sigma_z^{(A)}\sigma_z^{(B)} \tag{6.13}$$

In this framework, ZZ arises when the shared mode induces state-dependent frequency shifts that depend jointly on both qubit states. Two standard routes exist.

Route 1: Bus-mediated conditional phase via virtual photons (fourth order). To fourth order in g/Δ, one obtains:

$$\zeta \sim \frac{g_A^2 g_B^2}{\Delta_A \Delta_B}\left(\frac{1}{\Delta_A} + \frac{1}{\Delta_B}\right) \tag{6.14}$$

up to order-unity factors depending on truncation and rotating-wave structure. The exact coefficient follows from extending the Schrieffer–Wolff expansion to fourth order; the scaling (6.14) is the essential design dependence.

Route 2: Parametric activation. Modulate one coupling $g_A(t)$ or detuning $\Delta_A(t)$ at frequency $|\omega_A - \omega_B|$ to selectively activate conditional phase accumulation. This yields a tunable $\zeta(t)$ while keeping population exchange suppressed.

In both cases, the physical interpretation is direct: the common Ψ-mode enforces a shared phase condition, and conditional phase is the accumulated mismatch when boundary loading depends on joint qubit configuration.

6.7 Entanglement as phase-locking: dynamical statement, not metaphor

Define the two-qubit coherence (off-diagonal element) in the computational basis:

$$C_{AB}(t) = \langle 00|\rho(t)|11\rangle \tag{6.15}$$

Entanglement requires maintenance of nonzero two-qubit coherences under interaction. Under the unitary generated by (6.8), C_{AB} evolves with a phase determined by nonlocal terms (exchange and/or ZZ).

Thus, *phase-locking* is mathematically the persistence of stable relative phase evolution in joint coherences, enforced by the shared Ψ-bus mode.

Operationally, the bus provides a common dynamical reference for the phases of both qubits, allowing joint coherences to remain well-defined over gate times.

6.8 Decoherence in the coupled system and gate fidelity condition

Include minimal dissipation:

- bus loss rate κ,
- qubit dephasing rates $\Gamma_{\varphi A}, \Gamma_{\varphi B}$,
- optional relaxation rates Γ_{1A}, Γ_{1B}.

A Lindblad model reads:

$$\begin{aligned}\dot{\rho} = & -i[H,\rho] + \kappa\,\mathcal{D}[a]\rho + \Gamma_{\varphi A}\,\mathcal{D}[\sigma_z^{(A)}]\rho + \Gamma_{\varphi B}\,\mathcal{D}[\sigma_z^{(B)}]\rho \\ & + \Gamma_{1A}\,\mathcal{D}[\sigma_-^{(A)}]\rho + \Gamma_{1B}\,\mathcal{D}[\sigma_-^{(B)}]\rho\end{aligned} \tag{6.16}$$

where

$$\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}.$$

In the dispersive regime, bus loss induces additional dephasing:

$$\Gamma_{\varphi A}^{(\mathrm{bus})} \sim \kappa \left(\frac{g_A}{\Delta_A}\right)^2 \bar{n}_c, \qquad \Gamma_{\varphi B}^{(\mathrm{bus})} \sim \kappa \left(\frac{g_B}{\Delta_B}\right)^2 \bar{n}_c \tag{6.17}$$

where $\bar{n}_c$ is the mean bus occupancy during the operation.

A necessary gate condition for an entangling operation of duration $t_g \sim \pi/(4J)$ is:

$$t_g \ll \min\left\{\Gamma_{\varphi A}^{-1}, \Gamma_{\varphi B}^{-1}, \kappa^{-1}\right\} \tag{6.18}$$

More concretely, using $t_g \sim 1/J$:

$$J \gg \Gamma_{\varphi A}, \Gamma_{\varphi B}, \kappa \tag{6.19}$$

This is an explicit inequality linking entanglement generation to phase stability.

7 Computational scaling: vacuum phase-space versus particle transport

This section formalizes "computational power" as a dynamical resource in a Ψ-mode network, derives implementability of quantum circuits, and gives explicit scaling bounds set by phase noise, geometry, and control bandwidth. No interpretive language is used.

7.1 Model of an n-qubit Ψ-network

Let the effective n-qubit state be $\rho(t)$ on $(\mathbb{C}^2)^{\otimes n}$. The coherent dynamics are generated by a time-dependent Hamiltonian:

$$\begin{aligned} H(t) = \sum_{j=1}^{n} \left(\frac{\omega_j(t)}{2} \sigma_z^{(j)} + \Omega_j(t) \sigma_x^{(j)} \right) + \sum_{j<k} J_{jk}(t)\, \sigma_x^{(j)} \sigma_x^{(k)} \\ + \sum_{j<k} \zeta_{jk}(t)\, \sigma_z^{(j)} \sigma_z^{(k)} \end{aligned} \tag{7.1}$$

Interpretation in the Quarkbase layer:

- $\omega_j(t)$: detuning of confined Ψ-mode j (boundary-condition control),
- $\Omega_j(t)$: coherent mixing drive (phase-controlled injection),
- $J_{jk}(t), \zeta_{jk}(t)$: shared Ψ-structure couplings (bus-mediated or direct overlap).

This Hamiltonian form is standard in quantum computation; here its coefficients are explicitly tied to Ψ confinement and control.

7.2 Universality (constructive)

A set of controls is universal if it can approximate any $U \in SU(2^n)$ to arbitrary accuracy. A sufficient constructive condition is:

- arbitrary single-qubit rotations on each qubit:

$$\{\sigma_x^{(j)}, \sigma_z^{(j)}\} \quad \forall j \tag{7.2}$$

- at least one entangling two-qubit interaction between connected pairs (e.g. $\sigma_z^{(j)}\sigma_z^{(k)}$ or $\sigma_x^{(j)}\sigma_x^{(k)}$).

From Lie algebra controllability:

$$\mathfrak{L} = \mathrm{Lie}\Big(\{i\sigma_x^{(j)}, i\sigma_z^{(j)}, i\sigma_z^{(j)}\sigma_z^{(k)}\}\Big) = \mathfrak{su}(2^n) \tag{7.3}$$

Thus the Ψ-network implements universal quantum computation provided the physical platform can realize (7.2) and at least one entangling term. Section 6 provides explicit derivations for entangling interactions from a shared Ψ-bus.

7.3 A physically grounded resource: coherent phase-volume

In this framework, the operative resource is not "Hilbert space size" but **maintained coherent phase relationships** under control and noise.

Define the **coherent phase-volume** (CPV) at time t as:

$$\mathrm{CPV}(t) = \sum_{j=1}^{n} w_j \, \mathcal{C}_j(t) + \sum_{j<k} w_{jk} \, \mathcal{C}_{jk}(t) \tag{7.4}$$

where:

- $\mathcal{C}_j$ measures single-qubit coherence:

$$\mathcal{C}_j(t) = \left| \rho_{01}^{(j)}(t) \right| \tag{7.5}$$

- $\mathcal{C}_{jk}$ measures two-qubit joint coherence relevant to entanglement:

$$\mathcal{C}_{jk}(t) = \left| \langle 00 | \rho^{(jk)}(t) | 11 \rangle \right| \tag{7.6}$$

Weights w_j, w_{jk} encode architecture relevance (connectivity, gate schedule). CPV is operational: it lower-bounds the ability to execute coherent gates and maintain algorithmic interference.

Under pure dephasing (Section 5), coherences decay exponentially; thus CPV provides a quantitative bridge from Ψ-phase stability to computational performance.

7.4 Noise model and scaling of coherence

Assume dominant noise is dephasing (phase diffusion), described by:

$$\begin{aligned} \dot{\rho} = & -i[H(t), \rho] + \sum_{j=1}^{n} \Gamma_{\varphi j} \, \mathcal{D}[\sigma_z^{(j)}]\rho \\ & + \sum_{j<k} \Gamma_{\varphi jk} \, \mathcal{D}[\sigma_z^{(j)} \sigma_z^{(k)}]\rho \end{aligned} \tag{7.7}$$

For a single qubit j, Eq. (5.8) gives:

$$\mathcal{C}_j(t) = \mathcal{C}_j(0) e^{-2\Gamma_{\varphi j} t} \tag{7.8}$$

For a two-qubit coherence $\langle 00|\rho|11\rangle$:

$$\mathcal{C}_{jk}(t) = \mathcal{C}_{jk}(0) \exp[-2(\Gamma_{\varphi j} + \Gamma_{\varphi k})t] \tag{7.9}$$

Thus CPV decays at a rate set by the sum of phase-noise rates of the participating degrees of freedom. As n grows, the number of coherences required by deep circuits increases, making CPV preservation the scaling bottleneck.

7.5 Gate time, circuit depth, and a necessary inequality

Let:

- t_g be a characteristic two-qubit gate time (e.g. $t_g \sim 1/J$ or $t_g \sim 1/\zeta$),
- D be the circuit depth (number of sequential gate layers),
- $T = D\,t_g$ be the total coherent evolution time.

A necessary condition for coherence above a target threshold $\epsilon \ll 1$ is:

$$e^{-2\Gamma_\varphi T} \gtrsim \epsilon \quad \Rightarrow \quad T \lesssim \frac{1}{2\Gamma_\varphi} \ln\!\left(\frac{1}{\epsilon}\right) \tag{7.10}$$

With $T = D\,t_g$ and $t_g \sim 1/J$:

$$D \lesssim \frac{J}{2\Gamma_\varphi} \ln\!\left(\frac{1}{\epsilon}\right) \tag{7.11}$$

This inequality is architecture-agnostic and expresses the scaling limit: deeper circuits require either faster entangling interactions or lower phase noise.

In the Quarkbase layer this reads: **computational depth is bounded by maintained Ψ-phase coherence**.

7.6 Comparison point: particle transport versus Ψ-phase operations

Classical irreversible computation has a minimal energetic cost per logically irreversible operation (Landauer). That principle is not invoked here rhetorically; only the operational distinction is used:

- particle-based switching requires dissipative relaxation to a stable state at each step,
- Ψ-phase operations are generated by a conservative Hamiltonian and require dissipation only at readout.

Therefore, for a fixed error threshold, an architecture operating primarily via coherent phase operations can execute more logical transformations per unit dissipated energy, provided Γ_φ is sufficiently low.

This follows directly from (7.1) being Hamiltonian and (7.7) showing that dissipation enters mainly through dephasing and measurement coupling.

7.7 Network geometry enters through coupling scalings

In a confined-mode architecture, couplings scale with geometry because overlaps and boundary conditions determine g, Δ, J, ζ. A generic scaling law for bus-mediated exchange in the dispersive regime is:

$$J_{jk} \sim \frac{g_j g_k}{\Delta} \tag{7.12}$$

If g_j decays with separation r as $g(r) \sim r^{-p}$ (or exponentially in a bandgap structure), then:

$$J(r) \sim r^{-2p} \qquad \text{(power-law case)} \tag{7.13}$$

Thus connectivity and layout impose explicit constraints on achievable J and, via (7.11), on feasible circuit depth. This is the formal route by which "geometry dominates scaling" becomes a theorem-level statement.

8 Graphene as a boundary condition for Ψ and the Ψ-resonant qubit architecture

This section introduces an explicit boundary model for a two-dimensional crystal acting on the Ψ-field, derives the confined-mode spectrum, constructs a dual-port phase-controlled Ψ-qubit, and derives quantitative signatures: phase-dependent non-additivity and phase-controlled switching at constant total power.

8.1 Effective boundary model for a two-dimensional sheet

Let $\Psi(\mathbf{x}, t)$ satisfy the bulk equation (2.1) in $\mathbb{R}^3$, with a two-dimensional sheet located at $z = 0$. Denote $\mathbf{r} = (x, y)$.

We model the sheet as a singular boundary term in the action:

$$S[\Psi] = \int d^4x \left[\frac{1}{2}\left(\frac{1}{c_\Psi^2}(\partial_t\Psi)^2 - |\nabla\Psi|^2\right) - V(\Psi)\right] - \frac{1}{2}\int dt\, d^2r\, \kappa_s\, \Psi(\mathbf{r}, 0, t)^2 \tag{8.1}$$

Here $\kappa_s \geq 0$ is an effective *pinning strength* of the vacuum mode induced by the two-dimensional crystal.

Varying S yields the bulk equation together with a jump condition at $z = 0$. In the linear regime $V(\Psi) \approx \frac{1}{2}m_\Psi^2\Psi^2$, the field equation becomes:

$$\left(\frac{1}{c_\Psi^2}\partial_t^2 - \nabla^2 + m_\Psi^2\right)\Psi + \kappa_s\,\delta(z)\,\Psi(\mathbf{r}, 0, t) = 0 \tag{8.2}$$

Integrating (8.2) across an infinitesimal slab $(-0^+, +0^+)$ yields the discontinuity of the normal derivative:

$$\partial_z\Psi\big|_{0^+} - \partial_z\Psi\big|_{0^-} = \kappa_s\,\Psi(\mathbf{r}, 0, t) \tag{8.3}$$

Equation (8.3) is the precise mathematical meaning of a *two-dimensional sheet as a Ψ-boundary condition.*

8.2 Confined mode between two graphene sheets (vertical cavity)

Place two sheets at $z = 0$ and $z = L$ with pinning strengths κ_{s1} and κ_{s2}. Seek separable modes:

$$\Psi(\mathbf{r}, z, t) = u(z)\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \tag{8.4}$$

In the bulk, $u(z)$ satisfies:

$$-u''(z) + \left(k^2 + m_\Psi^2 - \frac{\omega^2}{c_\Psi^2}\right) u(z) = 0 \tag{8.5}$$

For confined standing waves define:

$$q^2 = \frac{\omega^2}{c_\Psi^2} - k^2 - m_\Psi^2, \qquad q > 0 \tag{8.6}$$

so that

$$u(z) = A\sin(qz) + B\cos(qz).$$

Applying the jump conditions at each sheet is equivalent (for a symmetric cavity) to Robin boundary conditions:

$$u'(0) = \frac{\kappa_{s1}}{2}u(0), \qquad u'(L) = -\frac{\kappa_{s2}}{2}u(L) \tag{8.7}$$

Solving (8.5) with (8.7) yields the quantization condition:

$$\tan(qL) = \frac{q(\kappa_{s1} + \kappa_{s2})}{2q^2 - \frac{\kappa_{s1}\kappa_{s2}}{2}} \tag{8.8}$$

In the strong-pinning limit $\kappa_{s1}, \kappa_{s2} \to \infty$, this reduces to:

$$q_n \approx \frac{n\pi}{L}, \qquad n = 1, 2, \ldots \tag{8.9}$$

The corresponding resonance frequencies are:

$$\omega_n^2 \approx c_\Psi^2 \left(k^2 + m_\Psi^2 + \left(\frac{n\pi}{L}\right)^2\right) \tag{8.10}$$

This yields an explicit geometry-controlled resonance family, consistent with the design relation $\omega_\Psi \propto 1/L$ when k and m_Ψ are negligible.

8.3 Planar (in-plane) graphene cavity: strip or patch resonator

For on-chip architectures, it is often more practical to confine the effective Ψ-mode in-plane, with characteristic length L and width W. The effective mode equation becomes:

$$\left(\frac{1}{c_\Psi^2}\partial_t^2 - \nabla_\parallel^2 + \mu_{\text{eff}}^2\right) \psi(\mathbf{r}, t) = 0 \tag{8.11}$$

where $\psi(\mathbf{r}, t) = \Psi(\mathbf{r}, z = 0, t)$ and μ_{eff} absorbs vertical confinement contributions.
With strong pinning $\psi = 0$ at $x = 0, L$, the fundamental mode has:

$$\omega_\Psi \approx c_\Psi \sqrt{\mu_{\text{eff}}^2 + \left(\frac{\pi}{L}\right)^2} \tag{8.12}$$

In the regime $\mu_{\text{eff}} \ll \pi/L$, this reduces to:

$$\omega_\Psi \approx \frac{\pi c_\Psi}{L} \tag{8.13}$$

Equations (8.10)–(8.13) provide the explicit spectral dependence required for design and falsifiability.

8.4 Dual-port coherent drive: input–output formulation

Let the Ψ-cavity be driven by two coherent ports $p = 1, 2$, both at angular frequency ω, with complex drive amplitudes:

$$s_1 = s\, e^{i\phi_1}, \qquad s_2 = s\, e^{i\phi_2}, \qquad \Delta\phi = \phi_2 - \phi_1 \tag{8.14}$$

Let $a(t)$ be the complex envelope of the cavity mode. In standard input–output form:

$$\dot{a} = \left(-i\Delta - \frac{\kappa}{2}\right) a - i\kappa_3 |a|^2 a + \sqrt{\kappa_1}\, s_1 + \sqrt{\kappa_2}\, s_2 \tag{8.15}$$

where:

- $\Delta = \omega - \omega_\Psi$,
- $\kappa = \kappa_1 + \kappa_2 + \kappa_{\text{int}}$,
- κ_3 is the effective Kerr nonlinearity inherited from Ψ self-interaction.

Define the coherent drive sum:

$$S(\Delta\phi) = \sqrt{\kappa_1}\, s\, e^{i\phi_1} + \sqrt{\kappa_2}\, s\, e^{i\phi_2} \tag{8.16}$$

If $\kappa_1 = \kappa_2 = \kappa_e$, then:

$$S(\Delta\phi) = 2\sqrt{\kappa_e}\, s\, e^{i(\phi_1 + \Delta\phi/2)} \cos(\Delta\phi/2) \tag{8.17}$$

Thus the effective injected field depends on $\cos(\Delta\phi/2)$ even when the per-port power $|s|^2$ is unchanged.

To enforce fixed total injected power $P_{\text{tot}} = |s_1|^2 + |s_2|^2 = 2s^2$, one keeps $|s_1| = |s_2| = s$ constant while varying $\Delta\phi$. This is the mathematically clean *constant-power, variable-phase* condition.

8.5 Steady-state amplitude equation and bistability

In steady state $\dot{a} = 0$, write $a = Ae^{i\theta}$. From (8.15):

$$\left[i\Delta + \frac{\kappa}{2} + i\kappa_3 A^2\right] Ae^{i\theta} = S(\Delta\phi)$$

Taking the modulus squared yields:

$$A^2 \left[\left(\Delta + \kappa_3 A^2\right)^2 + \left(\frac{\kappa}{2}\right)^2\right] = |S(\Delta\phi)|^2 \tag{8.18}$$

Because $|S(\Delta\phi)|^2 \propto \cos^2(\Delta\phi/2)$, the steady-state amplitude is multivalued near resonance for sufficiently strong κ_3.

The turning points satisfy $d|S|^2/dA = 0$, yielding the bistability condition:

$$\kappa_3 A^2 \gtrsim \frac{\kappa}{\sqrt{3}}, \qquad \text{near } \Delta \approx -\kappa_3 A^2 \tag{8.19}$$

This provides an explicit inequality for the existence of two stable amplitudes A_- and A_+.

8.6 Non-additivity and the differential observable

Define an observable proportional to intracavity energy:

$$R \propto A^2 \tag{8.20}$$

Measure three configurations:

- port 1 only: $S = S_1 = \sqrt{\kappa_1} s_1 \Rightarrow A_1$,
- port 2 only: $S = S_2 = \sqrt{\kappa_2} s_2 \Rightarrow A_2$,
- both ports coherent: $S = S(\Delta\phi) \Rightarrow A_{12}(\Delta\phi)$.

Define the differential response:

$$\Delta R(\Delta\phi) = R_{12}(\Delta\phi) - \Big(R_1 + R_2\Big) \tag{8.21}$$

In the nonlinear regime, ΔR becomes a sharply resonant, phase-dependent function derived directly from (8.18)–(8.21).

8.7 Phase-controlled switching at constant total power

With $P_{\text{tot}} = 2s^2$ fixed, sweeping $\Delta\phi$ yields:

$$|S|^2_{\max} = 4\kappa_e s^2 \quad (\Delta\phi = 0), \qquad |S|^2_{\min} = 0 \quad (\Delta\phi = \pi) \tag{8.22}$$

If (8.19) is satisfied, the system admits two stable solutions over a range of $\Delta\phi$. Sweeping $\Delta\phi$ produces hysteresis and switching between branches at critical phases $\Delta\phi_\uparrow$ and $\Delta\phi_\downarrow$.

Switching is controlled by phase alone, at constant total injected power, and localized near resonance.

8.8 Identification of qubit states in this architecture

Define:

$$|0\rangle \equiv \text{low-amplitude stable solution } A_-(\Delta\phi),$$

$$|1\rangle \equiv \text{high-amplitude stable solution } A_+(\Delta\phi).$$

Readout is performed by measuring $R \propto A^2$ (or any monotonic proxy), while control is achieved via $\Delta\phi(t)$ and detuning $\Delta(t)$.

8.9 Closed statement of this section

We have provided:

1. an explicit boundary-action model for a two-dimensional sheet yielding the jump condition (8.3),
2. confined-mode spectra for vertical and planar cavities (8.8)–(8.13),

3. a dual-port driven nonlinear cavity equation (8.15),

4. a closed-form steady-state relation (8.18), bistability condition (8.19), and measurable signatures: non-additivity (8.21) and phase-controlled switching at constant total power (8.22).

This completes the device-level mathematical core.

9 Noise, stability, and error mechanisms specific to Ψ-resonant graphene qubits

This section derives error channels from explicit noise sources acting on the dual-port Ψ-resonant qubit. We compute dephasing from phase and detuning noise, switching (bit-flip) errors from noise-activated escape in a bistable potential, and connect rates to the depth bound of Section 7.

9.1 Noise sources and parametrization

We consider three dominant noise processes relevant to the architecture of the previous section.

N1. Drive phase noise

$$\Delta\phi(t) = \Delta\phi_0 + \xi_\phi(t), \qquad \langle \xi_\phi(t)\xi_\phi(0)\rangle = 2D_\phi\,\delta(t) \tag{9.1}$$

N2. Frequency (detuning) noise

$$\Delta(t) = \Delta_0 + \xi_\Delta(t), \qquad \langle \xi_\Delta(t)\xi_\Delta(0)\rangle = 2D_\Delta\,\delta(t) \tag{9.2}$$

N3. Amplitude noise Included for completeness as fluctuations of $|S|^2$; neglected at leading order when total injected power is stabilized.

White-noise (Markovian) assumptions are sufficient to derive closed-form rates. Colored spectra can be included by replacing diffusion constants with spectral densities evaluated at zero frequency.

9.2 Effective potential for the bistable amplitude

From the steady-state equation (8.18), define the effective potential $U(A)$ for the amplitude dynamics via an overdamped envelope equation (valid when κ is the fastest scale for amplitude relaxation):

$$\dot{A} = -\frac{\partial U}{\partial A} + \eta_A(t) \tag{9.3}$$

A convenient form of the potential (up to an additive constant) is obtained by integrating the force implied by (8.18):

$$U(A) = \frac{1}{2}\left(\Delta + \kappa_3 A^2\right)^2 A^2 + \frac{\kappa^2}{8}A^2 - |S(\Delta\phi)|^2 \ln A \tag{9.4}$$

The minima of $U(A)$ correspond to the stable solutions $A_{\pm}$. The barrier height ΔU between minima determines the switching (bit-flip) rate.

9.3 Phase-noise-induced dephasing (pure dephasing)

Phase noise $\xi_\phi(t)$ enters the drive term through $|S(\Delta\phi)|^2 \propto \cos^2(\Delta\phi/2)$. Linearizing near an operating point $\Delta\phi_0$:

$$|S|^2 \approx |S_0|^2 \left[1 - \tan\left(\frac{\Delta\phi_0}{2}\right) \xi_\phi(t)\right] \tag{9.5}$$

Projecting onto the two-level manifold, fluctuations modulate the energy splitting $\varepsilon = E_1 - E_0$. To leading order:

$$\dot{\theta} = \varepsilon + \delta\varepsilon(t), \qquad \langle \delta\varepsilon(t)\delta\varepsilon(0)\rangle = 2D_\varepsilon\,\delta(t) \tag{9.6}$$

with

$$D_\varepsilon \propto \left(\frac{\partial\varepsilon}{\partial|S|^2}\right)^2 D_\phi \tag{9.7}$$

The off-diagonal coherence decays as:

$$\langle e^{i\theta(t)}\rangle = e^{-D_\varepsilon t} \quad \Rightarrow \quad \Gamma_\varphi^{(\phi)} = D_\varepsilon \tag{9.8}$$

Thus **pure dephasing is set by drive phase noise**, consistent with the architectural premise.

9.4 Detuning-noise-induced dephasing

Detuning noise $\xi_\Delta(t)$ enters additively in (8.18). Linearization yields:

$$\delta\varepsilon(t) \approx \left(\frac{\partial\varepsilon}{\partial\Delta}\right) \xi_\Delta(t) \tag{9.9}$$

Hence:

$$\Gamma_\varphi^{(\Delta)} = \left(\frac{\partial\varepsilon}{\partial\Delta}\right)^2 D_\Delta \tag{9.10}$$

Operation at a *sweet spot* where $\partial\varepsilon/\partial\Delta = 0$ minimizes this contribution; such points exist near symmetric operating conditions in the bistable regime.

9.5 Total dephasing rate

Assuming independent noise sources:

$$\Gamma_\varphi = \Gamma_\varphi^{(\phi)} + \Gamma_\varphi^{(\Delta)} + \Gamma_\varphi^{(\mathrm{bus})} \tag{9.11}$$

where $\Gamma_\varphi^{(\mathrm{bus})}$ is the coupling-induced contribution derived in (6.17).

9.6 Bit-flip errors: noise-activated switching (Kramers rate)

Amplitude fluctuations $\eta_A(t)$ induce transitions between the two stable minima A_- and A_+. In the overdamped regime, the Kramers escape rate is:

$$\Gamma_{\mathrm{bf}} \approx \frac{\omega_0}{2\pi} \exp\left(-\frac{\Delta U}{D_A}\right) \tag{9.12}$$

where:

- ω_0 is the small-oscillation frequency at the well minimum,
- ΔU is the barrier height from (9.4),
- D_A is the effective amplitude-noise diffusion constant.

Key consequences:

- Γ_{bf} is **exponentially suppressed** by increasing ΔU,
- bit-flip errors are negligible compared to dephasing in the deep bistable regime.

9.7 Error rates during phase-controlled switching

During intentional switching (writing operations), $\Delta\phi(t)$ is swept across a critical value. Non-adiabatic errors occur if the sweep rate exceeds the inverse relaxation time of A:

$$\dot{\Delta\phi} \ll \kappa \tag{9.13}$$

This inequality ensures deterministic switching with minimal overshoot and suppresses stochastic hopping.

9.8 Connection to circuit-depth bound

Using the total dephasing rate (9.11), the depth bound (7.11) becomes:

$$D \lesssim \frac{J}{2\left(\Gamma_\varphi^{(\phi)} + \Gamma_\varphi^{(\Delta)} + \Gamma_\varphi^{(\mathrm{bus})}\right)} \ln\left(\frac{1}{\epsilon}\right) \tag{9.14}$$

This expression explicitly links:

- control-electronics phase noise (D_ϕ),
- device geometry (through $\partial\varepsilon/\partial|S|^2$ and ΔU),
- bus coupling ($\Gamma_\varphi^{(\mathrm{bus})}$),

to achievable computational depth.

9.9 Design implications (formal)

From (9.8)–(9.14), a sufficient design strategy follows:

1. minimize D_ϕ via phase-locked sources and short path lengths,
2. operate at detuning sweet spots to suppress $\Gamma_\varphi^{(\Delta)}$,
3. increase ΔU (via κ_3 and operating phase) to exponentially suppress Γ_{bf},
4. keep J large relative to all dephasing channels.

These implications are mathematical consequences, not heuristics.

10 Experimental protocols and falsification criteria

This section specifies **minimal, reproducible experimental protocols** and **binary falsification criteria**. Each protocol is designed so that failure has a clear interpretation (model invalid), and success cannot be explained by linear cavities or generic electronic nonlinearities.

10.1 Device prerequisites (explicit)

- **Structure**: graphene (or equivalent two-dimensional crystal) patterned as a planar cavity of length L (strip or patch), with two symmetric coupling ports.
- **Encapsulation**: h-BN or equivalent to suppress charge traps.
- **Control**: single coherent source split into two arms with adjustable relative phase $\Delta\phi$; equalized amplitudes to keep P_{tot} constant.
- **Readout**: any monotonic proxy of intracavity energy $R \propto A^2$ (bolometric, thermal, noise power, reflectometry).
- **Calibration**: independent measurement of $\kappa, \kappa_{1,2}$, and confirmation of weak Kerr nonlinearity $\kappa_3 > 0$.

No quantum-limited measurement is required.

10.2 Protocol P1 — Geometry-controlled resonance

Procedure

1. Fabricate devices with varying cavity length L (same width W).
2. Measure the resonance $\omega_\Psi(L)$ at low drive (linear regime).
3. Fit to
$$\omega_\Psi(L) \approx \frac{\pi c_\Psi}{L} \quad \text{or} \quad \omega_\Psi^2(L) \approx c_\Psi^2 \left(\mu_{\mathrm{eff}}^2 + \frac{\pi^2}{L^2}\right)$$
(Eqs. 8.12–8.13).

Acceptance

- Monotonic inverse scaling with L within experimental uncertainty.

Falsification

- Resonance independent of L or dominated by carrier density with no geometric scaling.

10.3 Protocol P2 — Phase-controlled switching at constant power

Procedure

1. Fix $\omega \approx \omega_\Psi$.
2. Set equal port amplitudes $|s_1| = |s_2| = s$ so that $P_{\mathrm{tot}} = 2s^2$ is constant.
3. Sweep $\Delta\phi \in [0, 2\pi]$.
4. Record $R(\Delta\phi)$.

Prediction

- Bistable branches $A_\pm$ appear near resonance when (8.19) holds.
- Hysteresis in $R(\Delta\phi)$ with switching at critical phases $\Delta\phi_{\uparrow,\downarrow}$.

Falsification

- No switching with $\Delta\phi$ at constant P_{tot}.
- Switching that persists far off-resonance.

10.4 Protocol P3 — Resonant non-additivity

Procedure

1. Measure $R_1(\omega)$ with port 1 only.
2. Measure $R_2(\omega)$ with port 2 only.
3. Measure $R_{12}(\omega, \Delta\phi)$ with both ports at fixed P_{tot}.
4. Compute
$$\Delta R(\omega, \Delta\phi) = R_{12} - \Big(R_1 + R_2\Big).$$

Prediction

- $\Delta R \neq 0$ only within a narrow band around ω_Ψ.
- ΔR depends on $\Delta\phi$ and vanishes off-resonance.

Falsification

- ΔR independent of $\Delta\phi$.
- ΔR persists broadly across frequency (generic nonlinearity).

10.5 Protocol P4 — Phase-noise-limited coherence

Procedure

1. Inject controlled phase noise with known diffusion constant D_ϕ.
2. Measure decay of an off-diagonal coherence proxy (e.g. interference contrast, Ramsey-type envelope if available).
3. Extract Γ_φ.

Prediction

$$\Gamma_\varphi \propto D_\phi$$

(Eq. 9.8), with weak dependence on carrier mobility.

Falsification

- Decoherence dominated by scattering or mobility with negligible sensitivity to phase noise.

10.6 Protocol P5 — Detuning sweet spots

Procedure

1. Sweep detuning Δ while monitoring the dephasing rate.
2. Identify points where $\partial\varepsilon/\partial\Delta = 0$.

Prediction

- Local minima of $\Gamma_\varphi^{(\Delta)}$ (Eq. 9.10).

Falsification

- No detuning dependence of dephasing.

10.7 Control experiments (mandatory)

- **Incoherent drives**: decorrelate the two sources; all phase-dependent effects vanish.
- **Linear regime**: reduce drive so that $\kappa_3 A^2 \ll \kappa$; bistability and ΔR vanish.
- **Asymmetric ports**: break symmetry; shift thresholds consistent with (8.17).

Failure of these controls invalidates positive claims.

10.8 Binary decision table

Observation	Outcome
Phase-only switching at constant P_{tot} near ω_Ψ	Supports Ψ-resonant model
No phase dependence	Falsifies
$\Delta R(\omega, \Delta\phi)$ sharply resonant	Supports
Broad or non-resonant nonlinearity	Falsifies
$\Gamma_\varphi \propto D_\phi$	Supports
Γ_φ mobility-dominated	Falsifies

11 Mathematical consistency with the quantum formalism

This section proves that the reduced Ψ-dynamics constructed in Sections 3–10 are **mathematically equivalent** to standard qubit dynamics **within a well-defined regime**. The goal is not reinterpretation but equivalence: identical predictions for all operational quantities used in quantum computation.

11.1 Statement of equivalence

Claim. There exists a regime $\mathcal{R}$ of parameters and timescales in which the reduced dynamics of the confined Ψ-field are isomorphic to unitary evolution on $\mathbb{C}^{2^n}$ with standard measurement statistics.

We specify $\mathcal{R}$ explicitly below and prove equivalence constructively.

11.2 Reduced state space and mapping

From Section 3, the single-qubit reduced state is:

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle, \qquad |c_0|^2 + |c_1|^2 = 1 \tag{11.1}$$

Define the mapping:

$$\mathcal{M}: \quad (c_0, c_1) \mapsto \rho = |\psi\rangle\langle\psi| \tag{11.2}$$

This is the standard pure-state density operator. The mapping is injective up to a global phase.

11.3 Generator of evolution

From Sections 4 and 6, the reduced dynamics obey:

$$i\dot{\mathbf{c}} = H_{\text{eff}}(t)\mathbf{c} \tag{11.3}$$

with $H_{\text{eff}}(t)$ Hermitian. Therefore:

$$\mathbf{c}(t) = U(t)\mathbf{c}(0), \qquad U(t) = \mathcal{T}\exp\Big(-i\int_0^t H_{\text{eff}}(t')\,dt'\Big) \tag{11.4}$$

This is identical to the Schrödinger equation for a qubit.

11.4 Expectation values

For any Pauli operator σ_α, define the observable in the Ψ-reduced model by:

$$\langle \sigma_\alpha \rangle_\Psi = \mathbf{c}^\dagger \sigma_\alpha \mathbf{c} \tag{11.5}$$

This equals:

$$\langle \sigma_\alpha \rangle_{\mathrm{QM}} = \mathrm{Tr}(\rho\, \sigma_\alpha) \tag{11.6}$$

Thus all single-qubit expectation values coincide.

11.5 Multi-qubit extension

For n qubits, the reduced state vector $\mathbf{c} \in \mathbb{C}^{2^n}$ evolves under:

$$i\dot{\mathbf{c}} = H_{\mathrm{eff}}^{(n)}(t)\mathbf{c} \tag{11.7}$$

where $H_{\mathrm{eff}}^{(n)}$ is constructed from single-qubit and two-qubit terms derived in Sections 6–7.

Define:

$$\rho = |\psi\rangle\langle\psi| \tag{11.8}$$

All expectation values of products of Pauli operators satisfy:

$$\langle \sigma_{\alpha_1} \otimes \cdots \otimes \sigma_{\alpha_n} \rangle_\Psi = \mathrm{Tr}(\rho\, \sigma_{\alpha_1} \otimes \cdots \otimes \sigma_{\alpha_n}) \tag{11.9}$$

This establishes equivalence for all observables used in quantum algorithms.

11.6 Measurement statistics (Born rule)

From Section 5, strong measurement in the computational basis yields:

$$\rho \longrightarrow \begin{pmatrix} |c_0|^2 & 0 \\ 0 & |c_1|^2 \end{pmatrix} \tag{11.10}$$

Thus the probability of outcome $|j\rangle$ is:

$$P(j) = |c_j|^2 \tag{11.11}$$

No additional assumptions are required: the Born rule emerges as the operational statistic of irreversible coupling to the bath.

11.7 Domain of validity $\mathcal{R}$

The equivalence holds provided:

1. **Two-level truncation** is valid:

$$\Delta E_{2\to 3} \gg k_B T,\ \hbar\Omega \tag{11.12}$$

2. **Weak dissipation during gates**:

$$t_g \ll \Gamma_\varphi^{-1},\ \Gamma_1^{-1} \tag{11.13}$$

3. **Markovian noise approximation** is adequate over gate times.

4. **No leakage** outside the invariant manifold during control pulses.

These are precisely the standard operating conditions assumed in quantum hardware; no additional restrictions are introduced.

11.8 What is not claimed

- No claim of equivalence beyond the reduced manifold $\mathcal{R}$.
- No claim about microscopic ontology of all quantum phenomena.
- No modification of the quantum formalism.

The equivalence is **operational and exact within scope**.

11.9 Closed statement of this section

We have proved that, within an explicitly defined regime, the Ψ-resonant framework yields **identical unitary dynamics, expectation values, and measurement statistics** as standard quantum mechanics for quantum computation.

12 Relation to existing quantum computing platforms

This section situates the Ψ-resonant framework relative to established quantum computing platforms. The goal is not reinterpretation of their entire physics, but identification of the **shared dynamical core** and the **specific differences** that make Ψ-resonant graphene devices diagnostically distinct.

12.1 Superconducting qubits (transmons, flux qubits)

Standard description
Superconducting qubits are described by circuit quantization of lumped elements, leading to Hamiltonians of the form:

$$H = 4E_C(n - n_g)^2 - E_J \cos\varphi \tag{12.1}$$

After truncation, this yields an effective two-level system with Kerr nonlinearity.
Mapping to the Ψ-framework

- The superconducting phase φ plays the role of a **collective phase variable**.
- The Josephson junction provides the **nonlinearity** required for bistability.

- The microwave cavity or transmission line provides a **shared resonant structure**.

Thus:

- the **effective Hamiltonian** coincides with the reduced Ψ-dynamics,
- but the **physical carrier** is obscured by circuit abstraction.

Key difference

- Superconductors are **bulk 3D systems** with internal screening.
- Vacuum boundary conditions are indirect and averaged.
- Phase noise arises from materials and circuitry, not from explicit phase engineering.

Ψ-resonant graphene devices differ by exposing geometry-controlled vacuum confinement directly.

12.2 Photonic qubits (linear optics, cavities)

Standard description
Photonic qubits use occupation or mode encoding of electromagnetic fields, governed by Maxwells equations in cavities or waveguides.
Mapping

- The electromagnetic field is a **phase-coherent wave system**.
- Nonlinearity is either absent (linear optics) or weak (Kerr media).
- Entanglement arises via measurement-induced effects or engineered interactions.

In Ψ-language:

- photonic systems manipulate **transverse modes**,
- phase coherence is present,
- but **longitudinal vacuum coupling** is not directly accessed.

Key difference

- Without intrinsic nonlinearity, two-level truncation is artificial.
- Phase-controlled bistability at constant power is generally absent.

12.3 Spin qubits (quantum dots, NV centers)

Standard description
Spin qubits rely on discrete internal degrees of freedom of localized particles:

$$H = \frac{\omega}{2}\sigma_z + \mathbf{B}_{\text{drive}} \cdot \boldsymbol{\sigma} \tag{12.2}$$

Mapping

- The effective qubit algebra is identical.
- Coupling is mediated by exchange interactions or photons.

In Ψ-terms:

- the vacuum acts only as a **mediator**, not as the computational substrate,
- geometry plays a secondary role.

Key difference

- Scaling is limited by particle placement and variability.
- Phase coherence is local, not geometrically global.

12.4 Topological qubits

Standard description
Topological qubits rely on nonlocal encoding in collective modes protected by topology.
Mapping

- Protection is achieved by **global constraints**, not local energy gaps.
- Phase information is delocalized.

In the Ψ-framework:

- this corresponds to **global Ψ-phase constraints** enforced by geometry,
- protection arises from restricted phase-space access.

Key difference

- Topological platforms rely on exotic states of matter.
- Ψ-resonant platforms rely on **boundary-conditioned vacuum modes**.

12.5 What is common across all platforms

Across superconducting, photonic, spin, and topological platforms, the following elements are universal:

1. **Two-level truncation** of a larger dynamical system.
2. **Nonlinearity** enabling state selection.
3. **Phase coherence** as the critical resource.
4. **Shared resonant structures** for entanglement.
5. **Irreversible readout** via environment coupling.

These are precisely the elements derived from the Ψ-field framework.

12.6 What is specific to Ψ-resonant graphene devices

The Ψ-resonant graphene architecture uniquely offers:

- Direct geometric control of resonance frequency ($\omega_\Psi \propto 1/L$).
- Phase-only control of state switching at constant total power.
- Resonant, phase-dependent non-additivity as a diagnostic.
- Dominance of phase noise over scattering in decoherence.
- Minimal reliance on particle transport.

These features are not generic to other platforms and therefore serve as **falsifiable signatures**.

13 Implications for algorithms and computational paradigms

This section analyzes algorithmic structure **without rhetoric**, mapping standard quantum algorithms to Ψ-phase dynamics and identifying which algorithmic features are physically supported by this architecture and which are incidental.

13.1 Algorithmic primitives in physical terms

Any quantum algorithm can be decomposed into:

1. **State preparation**
2. **Unitary evolution (gates)**
3. **Interference**
4. **Measurement**

Within the Ψ-framework, these primitives correspond to:

- **Preparation** → initialization of a confined Ψ-mode in a stable basin ($|0\rangle$) or controlled superposition.
- **Gates** → controlled phase evolution and mode coupling (Sections 4–6).
- **Interference** → constructive/destructive phase alignment across paths.
- **Measurement** → irreversible phase decoherence via bath coupling (Section 5).

Thus, algorithmic execution is reducible to **phase trajectory engineering**.

13.2 Role of phase in quantum speedups

Consider a generic quantum algorithm expressed as:

$$|\psi_f\rangle = U_m U_{m-1} \cdots U_1 |\psi_0\rangle \tag{13.1}$$

The output probability for a measurement outcome x is:

$$P(x) = \left| \sum_{p \in \mathcal{P}_x} A_p e^{i\phi_p} \right|^2 \tag{13.2}$$

where p indexes computational paths contributing to outcome x.
In the Ψ-framework:

- amplitudes A_p correspond to **weights of coexisting Ψ-configurations**,
- phases ϕ_p correspond to **accumulated Ψ-phase** along trajectories in control-parameter space.

Quantum speedup arises when:

$$\phi_p - \phi_{p'} = 0 \pmod{2\pi} \quad \text{for desired paths,}$$

and destructive interference suppresses others.

This mechanism is **explicitly supported** by conservative Ψ-dynamics; no additional structure is required.

13.3 Grovers algorithm

Standard structure
Grovers search uses:

- uniform superposition,
- oracle phase inversion,
- diffusion operator (reflection about the mean).

Ψ interpretation

- Uniform superposition → symmetric Ψ-phase configuration across basis states.
- Oracle → localized phase shift on a subset of configurations.
- Diffusion → global phase rotation enforced by a shared Ψ-structure.

The quadratic speedup relies on:

- coherent phase alignment across iterations,
- not on population transfer per se.

Constraint
Grover iterations require $\mathcal{O}(\sqrt{N})$ coherent steps. From Section 7:

$$\sqrt{N} \lesssim \frac{J}{\Gamma_\varphi} \tag{13.3}$$

Thus, Grovers practical limit is **phase-coherence-limited**, consistent with observed scaling barriers.

13.4 Quantum Fourier Transform (QFT)

Standard structure
QFT relies on:

- controlled-phase gates with exponentially decreasing angles,
- global phase sensitivity.

Ψ interpretation

- Controlled-phase gates are natural in the Ψ-framework (Section 6).
- Small-angle phase rotations correspond to fine control of boundary conditions and drive phases.

Key implication
Precision of QFT scales with:

$$\delta\phi_{\min} \gtrsim \sqrt{D_\phi T} \tag{13.4}$$

Thus, achievable QFT depth is limited by **phase-noise spectral density**, not by qubit count alone.

13.5 Phase estimation and metrology

Phase estimation algorithms are **native** to the Ψ-framework because:

- the computational variable is phase,
- eigenphase accumulation corresponds to Ψ-phase winding.

In particular, the standard phase estimation circuit implements:

$$U|\psi\rangle = e^{i\theta}|\psi\rangle \quad \Rightarrow \quad \theta \leftrightarrow \oint \dot{\theta}(t)\, dt \tag{13.5}$$

This suggests Ψ-resonant devices are naturally suited for:

- analog phase estimation,
- hybrid computation–metrology tasks.

13.6 Algorithms less compatible with the architecture

Algorithms that rely primarily on:

- frequent mid-circuit measurements,
- rapid basis changes dominated by population transfer,
- heavy use of non-phase-diagonal noise models,

are less naturally supported, as each measurement incurs irreversible Ψ-decoherence. This is not a limitation of correctness but of **physical efficiency**.

13.7 Hybrid analog–digital paradigms

The Ψ-framework naturally supports **hybrid computation**:

- Analog phase evolution performs bulk interference.
- Digital control sequences discretize outcomes and manage branching.
- Measurement is deferred to the end to preserve coherence.

This aligns with variational algorithms and continuous-time quantum walks.

14 Information-theoretic interpretation and limits

This section formalizes information flow, storage, and loss in the Ψ-resonant framework using standard information-theoretic quantities. All results are derived from the dynamics already established; no new physical assumptions are introduced.

14.1 Information carriers in the Ψ-framework

In the reduced description, information is encoded in:

- **amplitudes** ($|c_j|$) (population weights),
- **relative phases** (θ_{jk}).

Define the reduced density matrix $\rho(t)$. All operational information accessible to computation is contained in ρ.

The key distinction is that:

- amplitudes are robust to pure dephasing,
- phases are fragile and carry the computational advantage.

14.2 Von Neumann entropy under Ψ-dynamics

The von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho) \tag{14.1}$$

Unitary Ψ-dynamics (Sections 4,6)

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] \quad \Rightarrow \quad \frac{dS}{dt} = 0 \tag{14.2}$$

Entropy is conserved. This reflects conservative Ψ-phase evolution.

Measurement / dephasing (Section 5)

Under dephasing:

$$\dot{\rho} = \Gamma_\varphi(\sigma_z \rho \sigma_z - \rho) \tag{14.3}$$

Entropy increases monotonically:

$$\frac{dS}{dt} \geq 0 \tag{14.4}$$

This increase is **not intrinsic** to computation but due to coupling with a macroscopic bath.

14.3 Mutual information and entanglement decay

For a bipartite system (AB), mutual information is:

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \tag{14.5}$$

Under unitary Ψ-dynamics, $I(A:B)$ is conserved when the subsystems are isolated.

Under local dephasing on A:

$$\dot{\rho}_{AB} = \Gamma_{\varphi A}(\sigma_z^{(A)} \rho_{AB} \sigma_z^{(A)} - \rho_{AB}) \tag{14.6}$$

One finds:

$$\frac{dI(A:B)}{dt} \leq 0 \tag{14.7}$$

Thus, entanglement and classical correlations decay due to **local phase diffusion**, consistent with the physical picture of phase information leaking into the environment.

14.4 Holevo bound and accessible information

Suppose a classical variable X is encoded in Ψ-states $\{\rho_x, p_x\}$. The accessible classical information is bounded by the Holevo quantity:

$$\chi = S\left(\sum_x p_x \rho_x\right) - \sum_x p_x S(\rho_x) \tag{14.8}$$

Under pure dephasing:

- $S(\rho_x)$ increases,
- coherences in $\sum_x p_x \rho_x$ decay.

Therefore $\chi(t)$ decreases monotonically. This formalizes the statement:

> Phase decoherence reduces the amount of information that can be extracted from interference.

14.5 Measurement cost and irreversibility

Measurement creates classical records. In the Ψ-framework this corresponds to irreversible transfer of phase information to the bath.

Let Y denote the measurement record. The entropy balance is:

$$\Delta S_{\text{sys}} + \Delta S_{\text{bath}} - \Delta I(\text{sys} : Y) \geq 0 \tag{14.9}$$

In projective measurement, $\Delta I(\text{sys} : Y)$ is maximized, and ΔS_{bath} compensates. This is consistent with Landauer-type bounds but does not require invoking them explicitly.

14.6 Information flow rate under phase noise

Define an **information coherence rate**:

$$\mathcal{R}_{\text{info}}(t) = -\frac{d}{dt} \sum_{j<k} |\rho_{jk}(t)|^2 \tag{14.10}$$

Under dephasing with rate Γ_φ:

$$\mathcal{R}_{\text{info}} = 4\Gamma_\varphi \sum_{j<k} |\rho_{jk}|^2 \tag{14.11}$$

This quantifies how fast usable interference information is lost to the environment.

14.7 Upper bound on algorithmic information extraction

Combining Section 7s depth bound with (14.11), the total extractable interference information in a computation of duration T is bounded by:

$$\int_0^T \mathcal{R}_{\text{info}}(t)\, dt \lesssim \sum_{j<k} |\rho_{jk}(0)|^2 \tag{14.12}$$

Thus:

- information is not created during computation,
- it is redistributed via unitary Ψ-dynamics,
- and destroyed by decoherence and measurement.

14.8 Closed statement of Section 14

We have shown that:

1. Unitary Ψ-dynamics conserve entropy and mutual information.
2. Dephasing irreversibly transfers phase information to the environment.
3. Standard information-theoretic bounds (Holevo, mutual information decay) follow directly.
4. Computational advantage corresponds to preserving phase information against decoherence.

15 Theoretical completeness and consistency checks

This section audits the framework for internal consistency, closure of assumptions, and absence of hidden gaps. Each potential objection is addressed with an explicit check or boundary condition.

15.1 Closure of dynamical assumptions

Assumption A1 (Physical Ψ-field). Introduced in Section 2 with an explicit action (2.1). **Check:** Dynamics are well posed (hyperbolic PDE), energy is bounded below with

$$V(\Psi) = \frac{1}{2} m_\Psi^2 \Psi^2 + \frac{\lambda}{4} \Psi^4,$$

and solutions exist globally for finite energy.

Assumption A2 (Weak nonlinearity). Used for bistability and two-level truncation (Section 3). **Check:** The perturbative regime is explicit:

$$\frac{\lambda \Gamma}{M} \ll \omega_0^2.$$

Outside this regime the truncation fails and is excluded from $\mathcal{R}$.

Assumption A3 (Single-/few-mode dominance). Used in Sections 3–9. **Check:** Valid when the mode spacing $\Delta\omega$ satisfies

$$\Delta\omega \gg \kappa, \Omega.$$

This is a standard cavity criterion and experimentally verifiable.

15.2 Consistency of reductions and truncations

Two-level truncation. Derived via bistability and invariant-manifold construction (Section 3). **Check:** Leakage probability to higher modes scales as

$$\left(\frac{\Omega}{\Delta E}\right)^2,$$

bounded by (11.12). This is identical to standard leakage bounds in circuit QED.

Rotating-wave and averaging approximations. Used in Sections 4 and 6. **Check:** Valid for

$$\omega \gg \kappa, \Omega, \Gamma_\varphi.$$

Corrections produce small Bloch–Siegert shifts, absorbed into renormalized parameters.

15.3 Conservation laws and symmetries

Energy conservation. Exact in isolated Ψ-dynamics (Sections 2–4). Broken only by explicit coupling to baths (Section 5), with rates accounted for.

Phase symmetry. Global phase invariance

$$\theta \to \theta + \text{const}$$

holds in the absence of drives. It is broken controllably by external coherent sources, as required for gates.

No hidden nonlocality. All couplings are mediated by explicitly modeled shared modes (Section 6). No instantaneous or acausal terms appear.

15.4 Measurement consistency

Measurement is implemented via a standard system–bath Hamiltonian (5.3) leading to Lindblad dynamics (5.4). **Check:** Complete positivity and trace preservation are guaranteed. Born-rule statistics emerge without extra postulates (11.11). No contradiction with standard measurement theory arises.

15.5 Information-theoretic consistency

Entropy production and mutual-information decay (Section 14) follow directly from Lindblad dynamics. **Check:** No violation of subadditivity, monotonicity, or Holevo bounds occurs. All inequalities are standard consequences of CPTP maps.

15.6 Parameter identifiability and experimental closure

Free parameters:

$$c_\Psi,\ m_\Psi,\ \lambda,\ \kappa_s,\ \kappa_3,\ \Gamma_\varphi,\ \kappa$$

Check: Each parameter is either:

- geometrically controlled ((L, W)),

- measurable via spectroscopy (ω_Ψ, κ),
- or extractable from response curves (bistability thresholds, noise scaling).

No parameter is left operationally undefined.

15.7 Limiting cases and consistency with known physics

- **Linear limit** ($\lambda \to 0$): recovers linear cavity physics; no qubit emerges (consistent).
- **Strong dissipation**: suppresses coherence; computation fails (consistent).
- **High temperature**: violates (11.12); computation fails (consistent).
- **No shared mode**: no entanglement (consistent).

In all limits, the framework reduces to known behavior without contradiction.

15.8 Absence of circular reasoning

No statement assumes quantum computation in order to explain quantum computation. The construction proceeds as:

Field dynamics $\to$ confinement $\to$ bistability $\to$ two-level reduction $\to$ unitary dynamics $\to$ meas

Each arrow is supported by explicit equations derived earlier.

16 Extensions and generalizations

This section extends the framework beyond two-level truncations, covering qudits, continuous-variable (CV) computation, and scalable mode networks. All extensions are derived from the same Ψ-field dynamics and inherit the same consistency conditions.

16.1 Beyond qubits: qudits from multi-stability

The Duffing-type nonlinearity in Section 3 yields bistability under weak drive. Stronger or engineered nonlinearities produce **multistability**.

Consider a generalized local potential:

$$V(\Psi) = \frac{1}{2} m_\Psi^2 \Psi^2 + \sum_{n=2}^{N} \frac{\lambda_{2n}}{2n} \Psi^{2n} \tag{16.1}$$

For suitable coefficients $\{\lambda_{2n}\}$, the effective amplitude equation admits $d > 2$ stable fixed points $\{A_0, \ldots, A_{d-1}\}$.

Define logical states:

$$|j\rangle \leftrightarrow A_j, \quad j = 0, \ldots, d-1 \tag{16.2}$$

The reduced state space is $\mathbb{C}^d$, with normalization inherited from bounded phase-space volume. The effective Hamiltonian becomes a $d \times d$ Hermitian operator constructed by projecting the envelope dynamics onto the invariant manifold spanned by $\{|j\rangle\}$.

Consistency check: leakage bounds generalize as

$$\Omega \ll \min_{j \neq k} \Delta E_{jk} \tag{16.3}$$

where ΔE_{jk} are energy separations between stable basins.

16.2 Qudit gates and universality

Single-qudit control is achieved by:

- detuning-induced diagonal generators (generalized Z),
- coherent mixing between selected pairs (generalized X) via phase-controlled drives.

A sufficient universality condition is the ability to generate the Lie algebra $\mathfrak{su}(d)$ on each site plus at least one entangling two-site interaction (e.g., generalized controlled-phase). This follows from standard controllability results and the same shared Ψ-bus mechanism used for qubits (Section 6).

16.3 Continuous-variable Ψ-computation

Dropping the two-level (or finite-level) truncation entirely yields **continuous-variable computation**.

Let the cavity envelope $a(t)$ evolve under:

$$\dot{a} = -i\Delta a - i\kappa_3 |a|^2 a - \frac{\kappa}{2} a + \sum_p \sqrt{\kappa_p}\, s_p(t) \tag{16.4}$$

This is a nonlinear CV system analogous to Kerr-based CV quantum computing.

CV primitives:

- Displacements: coherent drives $s_p(t)$.
- Phase rotations: detuning Δ.
- Non-Gaussianity: Kerr term $\kappa_3 |a|^2 a$.

Non-Gaussianity is necessary for CV universality; here it is intrinsic.

Noise model: phase diffusion and loss as in Sections 5 and 9 apply directly, yielding standard CV decoherence channels.

16.4 Hybrid qubit–CV architectures

Because both qubit (discrete) and CV regimes arise from the same Ψ-field, **hybrid architectures** are natural.

A common pattern:

- CV Ψ-modes perform analog interference and routing.
- Discrete qubits (two-level truncations) encode logical states and error correction.
- Measurement collapses CV states into discrete outcomes when needed.

Mathematically, this corresponds to block-structured Hamiltonians coupling finite-dimensional subspaces to oscillator modes, with coupling terms of the form already derived in Sections 6 and 8.

16.5 Mode networks and scalability

Consider a network of M Ψ-cavities with adjacency matrix $\mathcal{A}$. The linearized dynamics are:

$$\dot{\mathbf{a}} = -i\mathbf{K}\mathbf{a} - \frac{1}{2}\boldsymbol{\kappa}\mathbf{a} + \mathbf{S}(t) \tag{16.5}$$

where $\mathbf{K}$ encodes resonant frequencies and couplings.

Nonlinearity adds local Kerr terms:

$$(\dot{a}_j)_{\text{nl}} = -i\kappa_{3j}|a_j|^2 a_j \tag{16.6}$$

Scalability constraint: the eigenvalue spread of $\mathbf{K}$ must exceed dissipation to preserve mode distinguishability:

$$\min_{m\neq n} |\lambda_m - \lambda_n| \gg \max_j \kappa_j \tag{16.7}$$

Geometry (Section 8) controls $\mathbf{K}$, making scalability a layout problem rather than a materials problem.

16.6 Error correction beyond qubits

Error correction in this framework can exploit:

- **phase-space separation** (large ΔU) for bit-flip suppression,
- **geometric sweet spots** for dephasing suppression,
- **bosonic codes** in CV regimes (cat codes, binomial codes) implemented directly in Ψ-cavities.

These codes rely on the same Kerr nonlinearity and phase stability already present; no new physics is required.

17 Open parameters, experimental determination, and benchmarking

This section enumerates all remaining free parameters of the framework, specifies **direct experimental procedures** to determine each of them, and defines **benchmarking metrics** that allow quantitative comparison with existing quantum computing platforms. No parameter is left implicit.

17.1 Complete parameter list (closed)

From Sections 2–16, the framework depends on the following parameters:
Vacuum / Ψ-field

- c_Ψ: effective propagation speed
- m_Ψ: effective mass (or gap)

- λ: bulk nonlinearity coefficient

Boundary / device

- κ_s: 2D-sheet pinning strength
- (L, W): cavity geometry
- μ_{eff}: effective confinement mass (in-plane)

Cavity / mode

- ω_Ψ: resonance frequency
- κ: total linewidth
- $\kappa_{1,2}$: external coupling rates
- κ_3: Kerr (Duffing) nonlinearity

Control / noise

- D_ϕ: drive phase-noise diffusion constant
- D_Δ: detuning-noise diffusion constant
- Γ_φ: total dephasing rate
- Γ_1: relaxation rate (if present)

This list is exhaustive.

17.2 Determination of c_Ψ and m_Ψ

Protocol

1. Fabricate devices with multiple cavity lengths L_i.
2. Measure $\omega_\Psi(L_i)$ in the linear regime.
3. Fit to:

$$\omega_\Psi^2(L) = c_\Psi^2 \left(\mu_{\text{eff}}^2 + \frac{\pi^2}{L^2} \right) \tag{17.1}$$

Extraction

- Slope $\Rightarrow c_\Psi$
- Intercept $\Rightarrow c_\Psi \mu_{\text{eff}}$

If $\mu_{\text{eff}} \approx m_\Psi$ dominates, then m_Ψ is determined directly.

17.3 Determination of κ_s

From the jump condition (8.3), κ_s controls the deviation from ideal Dirichlet boundaries.

Protocol

- Compare measured $\omega_\Psi(L)$ to the ideal $\pi c_\Psi/L$.
- Fit to the full transcendental equation (8.8).

κ_s is extracted from deviations of q_n from integer multiples of π/L.

17.4 Determination of $\kappa, \kappa_{1,2}$

Standard cavity spectroscopy:

- Measure transmission or reflection $S_{21}(\omega)$.
- Fit linewidths and coupling asymmetries.

No model-specific assumptions are required.

17.5 Determination of κ_3 (nonlinearity)

Protocol

1. Drive near resonance with increasing power.
2. Measure resonance frequency shift $\delta\omega(A)$.

From Duffing theory:

$$\delta\omega \approx \kappa_3 A^2 \tag{17.2}$$

The slope of $\delta\omega$ versus A^2 yields κ_3.

17.6 Determination of dephasing parameters (D_ϕ, D_Δ)

Phase noise

- Inject calibrated phase noise.
- Measure coherence decay.
- Extract $\Gamma_\varphi^{(\phi)}$ and infer D_ϕ via (9.8).

Detuning noise

- Modulate gate voltage or magnetic bias with known noise.
- Measure induced dephasing.
- Extract D_Δ via (9.10).

17.7 Determination of Γ_1 (if present)

Measure relaxation by:

- preparing A_+,
- monitoring decay to A_- without sweeping $\Delta\phi$.

In the deep bistable regime, $\Gamma_1 \ll \Gamma_\varphi$; otherwise the qubit regime is invalid.

17.8 Benchmarking metrics (quantitative)

Define the following metrics:
Single-qubit

- Coherence time: $T_2 = 1/\Gamma_\varphi$
- Gate time: $t_g \sim 1/J$
- Quality factor: $Q = T_2/t_g$

Two-qubit

- Entangling rate: J or ζ
- Two-qubit fidelity (via randomized benchmarking)
- Crosstalk (off-resonant coupling)

Energy efficiency

- Dissipated energy per gate:

$$E_{\text{gate}} \sim \kappa \int_0^{t_g} A^2(t)\, dt \tag{17.3}$$

This quantity is architecture-independent and allows direct comparison to superconducting and photonic platforms.

17.9 Comparison baseline

For fair benchmarking:

- Normalize by gate fidelity (e.g. 99.9%).
- Compare Q, E_{gate}, and achievable depth D from (9.14).

This avoids qualitative claims and forces quantitative assessment.

18 Boundary of validity and potential failure modes

This section delineates **where the framework is valid**, **how it can fail**, and **how to distinguish genuine falsification from benign parameter drift**. Each failure mode is tied to a concrete observable.

18.1 Breakdown of the two-level truncation

Condition
The two-level reduction fails when higher modes become dynamically accessible:

$$\Omega,\ \kappa_3 A^2,\ k_B T\ \gtrsim\ \Delta E_{2\to 3} \tag{18.1}$$

Observable signatures

- Appearance of additional resonances in spectroscopy.
- Loss of bistability (single-valued amplitude response).
- Leakage evidenced by non-exponential coherence decay.

Interpretation
This is **not** a falsification of the Ψ-field model; it invalidates the *qubit* regime only. The correct description reverts to CV dynamics (Section 16).

18.2 Failure of single-/few-mode dominance

Condition
Mode spacing comparable to linewidth:

$$\Delta\omega \lesssim \kappa \tag{18.2}$$

Observable signatures

- Mode hybridization and avoided crossings.
- Sensitivity of response to small geometric disorder.
- Loss of clean phase-controlled switching.

Interpretation
This indicates insufficient confinement or over-coupling. The remedy is geometric (increase L-contrast or reduce κ), not theoretical.

18.3 Dominance of amplitude noise over phase noise

Condition
Amplitude diffusion exceeds phase diffusion:

$$D_A \gtrsim D_\phi \tag{18.3}$$

Observable signatures

- Random hopping between $A_\pm$ without phase sweeps.
- Bit-flip rates comparable to or exceeding dephasing rates.
- Breakdown of the dephasing-dominated error model.

Interpretation
The architecture no longer supports phase-centric computation. This constitutes a **practical failure** for computation, but not a falsification of the underlying Ψ dynamics.

18.4 Loss of conservative dynamics (excess dissipation)

Condition
Intrinsic dissipation competes with coherent evolution:

$$\kappa \gtrsim \omega_\Psi \quad \text{or} \quad \kappa \gtrsim J \tag{18.4}$$

Observable signatures

- Broad, overdamped resonances.
- Absence of hysteresis even at high drive.
- Strong heating correlated with response.

Interpretation
The assumption of near-conservative Ψ-dynamics fails. This is an **engineering limitation** (materials, contacts), not a theoretical inconsistency.

18.5 Breakdown of phase-only control

Condition
Relative phase $\Delta\phi$ is not the sole control variable:

$$\frac{\partial A}{\partial |S|^2} \sim \frac{\partial A}{\partial \Delta\phi} \tag{18.5}$$

Observable signatures

- Switching tracks amplitude noise more strongly than phase sweeps.
- Phase-controlled effects persist far off-resonance.

Interpretation
This falsifies the **specific Ψ-resonant qubit implementation** if persistent under controls (Section 10.7). It does not, by itself, falsify the Ψ-field hypothesis unless phase effects are absent *at resonance* under constant power.

18.6 Non-resonant nonlinearity masquerading as Ψ-effects

Condition
Generic electronic or thermal nonlinearities dominate:

- Joule heating,
- contact resistance changes,
- carrier density modulation.

Observable signatures

- Broad frequency response.

- Strong dependence on DC bias and temperature.
- Similar effects in incoherent-drive controls.

Discriminator
Repeat Protocols P2–P4 with:

- incoherent sources,
- detuned frequencies,
- reduced nonlinearity.

Persistence of effects only under coherent, resonant conditions is required.

18.7 Strong coupling to uncontrolled environments

Condition
Unmodeled baths introduce non-Markovian noise.
Observable signatures

- Non-exponential decoherence.
- History-dependent switching thresholds.
- Hysteresis unrelated to phase.

Interpretation
The Markovian approximation fails; extended bath modeling is required. This is a **model extension**, not a contradiction.

18.8 Genuine falsification criteria (hard)

The framework is **falsified** (not merely limited) if **all** of the following are satisfied:

1. Devices exhibit clear geometry-controlled resonances.
2. Nonlinearity is verified ($\kappa_3 > 0$).
3. Total injected power is held constant.
4. Relative phase $\Delta\phi$ is swept coherently.
5. Measurements are made near resonance.

AND the following are observed:

- No phase-dependent switching.
- No resonant non-additivity.
- Decoherence insensitive to phase noise.

Under these conditions, the central claim—phase-engineered Ψ-dynamics as the carrier—fails.

19 Synthesis and minimal claims

This section compresses the entire construction into **irreducible, theorem-like statements**, with explicit hypotheses and conclusions. No extensions, interpretation, or advocacy are included.

19.1 Definitions (fixed)

- **Ψ-field**: a physical, continuous scalar field obeying a conservative wave equation with weak local nonlinearity (Section 2).
- **Ψ-cavity**: a bounded region whose geometry and boundary conditions discretize Ψ-modes (Section 8).
- **Ψ-qubit**: a two-level invariant manifold arising from bistability of a confined Ψ-mode under coherent drive (Section 3).
- **Ψ-gate**: controlled unitary evolution generated by phase and detuning control of Ψ-modes (Sections 4, 6).
- **Measurement**: irreversible decoherence induced by coupling the Ψ-qubit to a macroscopic bath (Section 5).

These definitions are operational and mathematically specified.

19.2 Minimal hypotheses

The framework relies on the following hypotheses only:

H1. The vacuum supports a conservative field Ψ with well-posed dynamics.

H2. Boundary conditions and geometry discretize Ψ into confined modes.

H3. Weak nonlinearity produces bistability under coherent driving.

H4. Phase-coherent driving and shared modes enable controlled coupling.

H5. Measurement corresponds to irreversible system–bath coupling.

No hypothesis assumes quantum mechanics beyond its operational formalism.

19.3 Main results (formal)

Result R1 (Existence of a qubit). Under H1–H3 and appropriate parameter separation, a confined Ψ-mode admits a two-level invariant manifold with unitary reduced dynamics.

Established in Sections 2–4.

Result R2 (Gate equivalence). Phase and detuning control generate all single-qubit unitaries; shared Ψ-modes generate entangling two-qubit unitaries.

Established in Sections 4, 6, 7.

Result R3 (Measurement statistics). Coupling to a macroscopic bath yields Born-rule statistics for computational basis measurements.

Established in Sections 5 and 11.

Result R4 (Algorithmic support). Standard quantum algorithms relying on phase interference are supported within the coherence bounds set by phase noise and geometry.

Established in Sections 7, 13, 14.

Result R5 (Diagnostic signatures). Phase-controlled switching at constant total power and resonant non-additivity are necessary signatures of Ψ-resonant implementations.

Established in Sections 8–10.

19.4 Bounds and limits (explicit)

Coherence bound

$$T \lesssim \frac{1}{2\Gamma_\varphi} \ln\left(\frac{1}{\epsilon}\right)$$

Section 7.

Depth bound

$$D \lesssim \frac{J}{2\Gamma_\varphi} \ln\left(\frac{1}{\epsilon}\right)$$

Section 7.

Validity bound

$$\Omega,\ k_B T \ll \Delta E_{2\to 3}$$

Sections 11 and 18.

These bounds delimit applicability without ambiguity.

19.5 Non-claims (explicit exclusions)

The framework does **not** claim:

- modification of quantum mechanics,
- resolution of the general measurement problem,
- universal applicability beyond computation,
- superiority over all platforms independent of parameters.

Any such claim would exceed the proven scope.

19.6 Logical closure

All statements in Sections 1–18 reduce to:

$$\text{Conservative phase dynamics} \xrightarrow{\text{geometry + nonlinearity}} \text{two-level reduction} \xrightarrow{\text{control}} \text{quantum computation}$$

Each arrow is supported by explicit equations and experimentally accessible parameters.

20 Conclusions and outlook

This final section closes the work **without rhetoric** and specifies **only what is strictly implied** by the preceding analysis, plus **immediate next steps** that decide the framework empirically.

20.1 Conclusions (minimal, non-interpretive)

From Sections 1–19, the following conclusions are established within the stated domain:

1. **Quantum computation admits a fully physical implementation** based on conservative phase dynamics of a confined field.
2. **A two-level computational system (qubit)** arises from bistability of a driven, weakly nonlinear resonant mode under explicit, controllable conditions.
3. **Unitary gate dynamics** emerge from phase and detuning control, with no modification of the quantum formalism.
4. **Entanglement** is generated by shared resonant structures and is mathematically equivalent to standard two-qubit coupling mechanisms.
5. **Measurement statistics** follow from irreversible coupling to macroscopic degrees of freedom, yielding Born-rule outcomes.
6. **Scalability limits** are set by phase coherence, geometry, and coupling strength, not by abstract state-space size.
7. **Ψ-resonant graphene architectures** provide concrete, falsifiable experimental signatures not generic to other platforms.

These statements exhaust the proven content.

20.2 Immediate experimental priorities (ranked)

The framework reduces decisively to a small number of experiments:

Priority 1 — Phase-only control test
Demonstrate bistable switching controlled solely by relative phase at constant total injected power near resonance.

Priority 2 — Resonant non-additivity
Measure $\Delta R(\omega, \Delta\phi)$ and verify strict confinement to the resonance window.

Priority 3 — Phase-noise dominance
Quantify decoherence scaling with injected phase noise and suppress mobility-based explanations.

Priority 4 — Geometry scaling
Verify $\omega_\Psi \propto 1/L$ over multiple device lengths with fixed materials.

Each experiment is binary in outcome.

20.3 Decisive confirmation conditions

The framework is **confirmed at the operational level** if all of the following are observed in the same device class:

- Geometry-controlled resonant frequency.
- Verified weak Kerr nonlinearity.
- Phase-controlled switching at constant total power.
- Resonant, phase-dependent non-additivity.
- Decoherence dominated by phase noise.

No further interpretation is required.

20.4 Decisive refutation conditions

The framework is **refuted** if, under the controlled conditions above:

- Phase sweeps do not affect state selection.
- Non-additivity persists off-resonance or without coherence.
- Decoherence scales primarily with scattering or heating, independent of phase noise.

In that case, the Ψ-resonant hypothesis fails within its stated scope.

20.5 Outlook (restricted)

If confirmed, the framework implies:

- Quantum computation is an instance of **engineered vacuum phase dynamics**.
- Graphene and related 2D systems are **diagnostic**, not exclusive.
- Future progress depends on **phase control, geometry, and noise engineering**, not on interpretive advances.

If refuted, the failure is localized and instructive; no broader claims are affected.

21 Bibliography

References

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge (2010).

[2] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, Oxford University Press, Oxford (2002).

[3] C. W. Gardiner and P. Zoller, *Quantum Noise*, Springer, Berlin (2004).

[4] A. A. Clerk *et al.*, "Introduction to quantum noise, measurement, and amplification," Rev. Mod. Phys. **82**, 1155 (2010).

[5] A. Blais *et al.*, "Cavity quantum electrodynamics for superconducting electrical circuits," Phys. Rev. A **69**, 062320 (2004).

[6] J. Koch *et al.*, "Charge-insensitive qubit design derived from the Cooper pair box," Phys. Rev. A **76**, 042319 (2007).

[7] A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations*, Wiley, New York (1979).

[8] M. H. Devoret and J. M. Martinis, "Implementing qubits with superconducting integrated circuits," Quantum Inf. Process. **3**, 163 (2004).

[9] A. H. Castro Neto *et al.*, "The electronic properties of graphene," Rev. Mod. Phys. **81**, 109 (2009).

[10] D. F. Walls and G. J. Milburn, *Quantum Optics*, Springer, Berlin (2008).

[11] C. Omeñaca Prado, *Genesis Quarkbase: A New Genesis for Physics*, Zenodo (2025), doi:10.5281/zenodo.XXXXXXX.

[12] C. Omeñaca Prado, "The Next Electromagnetic Revolution: Maxwells Equations in the Framework of Quarkbase Cosmology," Zenodo (2025), doi:10.5281/zenodo.XXXXXXX.

[13] C. Omeñaca Prado, "Optical Absorption, Quantum Hall Effect, and Superconductivity in Graphene: An Interpretation from Quarkbase Cosmology," Zenodo (2025), doi:10.5281/zenodo.XXXXXXX.

[14] C. Omeñaca Prado, "A Quarkbase Cosmology Explanation of Superconductivity and Thermal Hyperconductivity in Graphene," Zenodo (2025), doi:10.5281/zenodo.XXXXXXX.